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Wm. E. Craighill
2nd Lt. Engrs.
May 7, '86

TO ACCOMPANY THIS VOLUME.

A REDUCTION DIAGRAM.

A LARGE diagram, 20 × 24 inches, finely engraved, has been prepared expressly to accompany this work. It takes the place of Table I., p. 103, and saves all multiplications. It is more accurate than the table for distances over 500 feet. It gives elevations to 50 feet (or yards or metres) for distances to 1,500 feet (or yards or metres), thus covering more than 95 per cent of all the cases occurring in ordinary topographical work. It also gives distance corrections to horizontal readings of less than 1,250 feet (or yards or metres), with vertical angles less than 15 degrees. The difference of elevation is read to the nearest tenth of a foot (or yard or metre) by reading the diagram to the nearest half-space, which is two millimetres square; while corrections to the horizontal distance for inclined sights are read to the nearest foot (or yard or metre) by reading to single spaces of two millimetres each.

The diagram is arranged for both distances and elevations taken in the same unit; and this may be 1 foot, 5 feet, 1 yard, or 1 metre, as is best adapted to the work in hand, and as the stadia rods are graduated.

Values are taken from it more accurately and with greater rapidity than from a table of equal compass, even were no multiplications to be made. The results are read from the diagram about as fast as an assistant can call off the arguments, and record the difference of elevation or corrected distance; both these results being taken from the same sheet. It has been used in practice, and found to be the most efficient means yet devised for reducing stadia notes. *A single reading of the diagram gives the desired final result.*

This diagram is printed on heavy lithographic paper, and will be found extremely accurate.

Sent postpaid, carefully packed in pasteboard rolls, for 50 cents each.

JOHN WILEY & SONS.

A MANUAL

OF

THE THEORY AND PRACTICE

OF

TOPOGRAPHICAL SURVEYING

BY MEANS OF

THE TRANSIT AND STADIA;

INCLUDING

SECONDARY BASE-LINE AND TRIANGULATION MEASUREMENTS, AND THE PROJECTION OF MAPS; ACCOMPANIED BY REDUCTION TABLES AND DIAGRAMS, PLATES OF MAP-LETTERING AND TOPOGRAPHICAL SIGNS.

BY

J. B. JOHNSON, C.E.,

PROFESSOR OF CIVIL ENGINEERING, WASHINGTON UNIVERSITY; FORMERLY ENGINEER ON THE UNITED STATES LAKE AND MISSISSIPPI RIVER SURVEYS.

Designed for the Use of Students and Engineers

IN THE CLASS-ROOM, FIELD, AND OFFICE.

NEW YORK:
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PREFACE.

FOR many years there has been a growing demand for more minute and exact information of a practical kind on the subject of Stadia Surveying. This method of topographical surveying began to receive attention in this country about twenty years ago. It has now come into very general use, notwithstanding the fact that the literature on the subject has been meagre and scattered, no adequate description of the field and office methods ever having been published in the English language. Reduction tables, or diagrams, also have generally been either wholly inaccessible, or were not adapted to the work in hand. It is hoped the accompanying manual may supply both these wants.

In the past eight years some five thousand square miles of country on the banks of the Mississippi River and its tributaries have been surveyed by this method; and on more than half of this work, five-foot contours have been carefully determined. On other government, state, and corporation surveys, fully twice as much more has been done, making a total area of about *fifteen thousand square miles of topography* surveyed in this country by means of the transit and stadia in the past twenty years. The method is constantly growing in favor, and is rapidly supplanting the use of the plane table.

The system is well adapted to preliminary railroad and canal surveys; surveys of drainage basins, reservoir, dam, and bridge

sites; the location of ditches and pipe lines; and, in fact, to surveys of any kind demanding a knowledge of the topographical features or of the contours of the ground. Where contours are to be determined with considerable accuracy over extended areas, there is no other method that will compare with it in cost and accuracy of results.

This system furnishes a solution of the now much mooted questions of railway location. This controversy turns mainly on the cost and accuracy of contour maps; cheap maps being worthless, and good maps costing too much time and labor. By the transit and stadia, very accurate contour maps can be procured at a moderate cost.*

The system would doubtless have come into much more general use in these various directions before now, if it had been presented to the profession in practicable shape for field and office use.

It is only a question of time before most of the States in the Union will inaugurate general topographical or geological surveys; and when this is done, the transit and stadia will doubtless be the instruments used. All young engineers should acquaint themselves with the method, and many engineers in practice would find it to their advantage to adopt the system in certain kinds of work.

The writer has three objects in view in the preparation of this work; viz., —

First, To prepare a manual that could profitably be put into the hands of students in surveying, and then be of further use to them in the field.

Second, To so clearly and minutely explain the theory and methods of field work, that an engineer in practice could, with-

* See correspondence on this subject in The Railroad Gazette, Nov. 14, 1884, to Jan. 9, 1885, with editorials on The Use and Abuse of Topography in the issues of Jan. 9 and March 6, 1885.

out other instruction, prepare his instruments, and do the work in good shape.

Third, To furnish such means of reducing the field notes, with such methods of plotting as have resulted from many years' experience of many engineers engaged in the business.

A reduction diagram has been prepared to accompany this manual, to be used in place of the table in finding distances and elevations from inclined sights. It is printed on heavy paper directly from the engraved plate, and gives elevations readily to the nearest tenth of a foot, and distances to fifteen hundred feet. It saves the multiplications otherwise required in the use of the table, and is quite as accurate.

A chapter is given on the "Projection of Maps," with tables for laying out a polyconic projection; also one on the measurement of base lines by a steel tape, and on methods of observation and reduction employed in triangulation schemes intended as a basis for topographical surveys.

If the writer may learn that he has even partially accomplished his objects, as above stated, and that he has contributed to a further adoption of this very efficient method of surveying, he will feel repaid for his labor.

J. B. J.

ST. LOUIS, April, 1885.

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TOPOGRAPHICAL SURVEYING

BY MEANS OF THE

TRANSIT AND STADIA.

INTRODUCTION.

1. For locating points on the surface of the ground in geographical position, as well as in elevation, there is, perhaps, no method equal to that of the transit and stadia. The transit may be any ordinary field transit with vertical circle; the stadia is a graduated rod or board. These are the only instruments necessary to make an elaborate topographical survey.

The principle of the location of points by this method, both horizontally and vertically, is that of polar co-ordinates; that is, the location of the point geographically is by obtaining its angular direction from the meridian through the instrument, which is read on the limb of the transit, and its distance from the instrument, which is read through the telescope on the stadia rod which is held at the point. This distance is found by observing what portion of the image of the graduated rod is included between certain cross hairs in the telescope. The farther the rod is from the instrument, the greater is the portion of the image which falls between the cross wires.

For elevation, the vertical angle is read on the vertical circle of the transit, when the telescope is directed towards a point

of the stadia rod as far from the ground as the telescope is above the stake over which it is set. The tangent of this angle of elevation, or depression, into the given horizontal distance, is the amount by which the point is above or below the instrument station.

In this way, both the chain and levelling-instrument are dispensed with, and the slow and laborious processes of chaining over bad ground, and levelling up and down hill, are avoided. The horizontal distances are obtained as well, in general, as by the chain; and the levelling may be done within a few tenths of a foot to the mile, which is amply sufficient for topographical purposes.

2. The stadia was first used in this way about the year 1820 by an Italian engineer. In 1836 it was introduced into the topographical and military survey of Switzerland. The stadia was first used in this country by Mr. J. R. Mayer, C.E., who brought the method with him from Switzerland.* This was in 1850; and, he having become connected with the United States Lake Survey, it was officially adopted on that service in 1864, and was thenceforward there exclusively used for topographical work. During the fourteen years that Mr. Mayer was connected with the Lake Survey (1862–76) he did much toward perfecting the method. It is now used on almost all topographical surveys made under the engineer corps of the army, and is being introduced into State geological and topographical surveys. It is also well suited to preliminary railroad surveys, to surveys for drainage and reservoir projects, and to all surveys for general mapping purposes, with or without contours. For all river, harbor, and coast work, it is especially adapted, and may even be used for land surveying. It replaces the plane table, and is capable of a more varied use.

* See Journal of the Franklin Institute for January, 1865.

CHAPTER I.

THEORY OF STADIA MEASUREMENTS.

I. *Fundamental Relations.*

1. In Fig. 1 let LS be any lens, or combination of lenses, used for the object glass of a telescope.

Let A_2B_2 be a portion of the object (in this case the stadia rod), and let A_1B_1 be its image. The point of the object A_2 has its image formed at A_1, and so with B_2 and B_1.

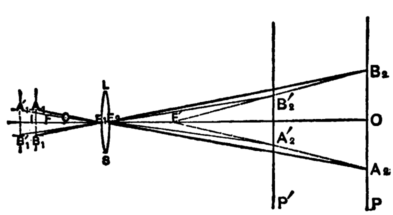

Fig. 1.

Let F be the position of the image for parallel rays, or for an object an infinite distance away; and let C be the centre of the instrument, or the intersection of the plumb line, extended, with the axis of the telescope.

Let E_1 and E_2 be the "principal points,"* and let the distance $FE_1 = f$ (focal length),

$$\left.\begin{array}{l} IE_1 = f_1 \\ OE_2 = f_2 \end{array}\right\} \text{(conjugate foci),}$$

$A_1B_1 = i$ (for image, intercepted portion),

$A_2B_2 = s$ (for stadia, intercepted portion).

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* As optics is generally taught in the English text-books, E_1 and E_2 are made to coincide in a point at or near the centre of the lens; and this is called the

Then, since A_1E_1 is parallel to A_2E_2, and B_1E_1 is parallel to B_2E_2, we have

$$A_1B_1 : A_2B_2 :: IE_1 : OE_2,$$

or,

$$i : s :: f_1 : f_2. \qquad (1)$$

Also, from the law of lenses we have

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}. \qquad (2)$$

2. On these two equations rests the whole theory of stadia measurements.

Since the distance $FE_1 = f =$ focal distance, is a constant for any lens or fixed combination of lenses, we see from equation (2), that, if the object P approaches the lens, the distance f_2 is diminished, and therefore f_1 must be increased; that is, the image recedes farther from the lens as the object approaches it, and *vice versa.*

If the extreme wires in the reticule of the telescope be supposed to be placed at A_1 and B_1 in the figure, then $A_1E_1B_1$ is the visual angle which is equal to $A_2E_2B_2$. But, as the image changes its distance from the objective as the object is nearer to or farther from the instrument, so the reticule is moved back and forth,* for it must always be in the plane of the image. Therefore $IE_1 = f_1$ is a variable quantity, while A_1B_1 is constant for fixed wires. *Therefore the visual angles at* E_1 *and* E_2 *are variable.*

If these angles were constant, the space intercepted on the rod, and the distance of the rod from the objective, would be in

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"optical centre." The "principal points" of the ordinary objective fall inside the surfaces of the lens, but they never coincide. The ordinary theory is sufficiently approximate for the development of stadia formula; but it saves confusion to make the conditions rigid, and it is equally simple.

* This discussion is worded for an *inverting*-telescope, since these are best adapted for stadia work.

constant ratio. Since this is not true, we must find the relation that does exist between the distance E_2O and the space intercepted on the rod, A_2B_2.

From equation (1) we have

$$\frac{1}{f_1} = \frac{s}{if_2},$$

but from equation (2),

$$\frac{1}{f_1} = \frac{1}{f} - \frac{1}{f_2}.$$

Equating these two values of $\frac{1}{f_1}$, we have

$$\frac{s}{if_2} = \frac{1}{f} - \frac{1}{f_2}$$

or

$$f_2 = \frac{f}{i}s + f; \tag{3}$$

that is, *the distance of the rod from the objective* is equal to the intercepted space in the rod multiplied by the constant ratio $\frac{f}{i}$, plus the constant f, where f is the focal length of the objective, and i is the distance between extreme wires. If the distance between the extreme wires be made 0.01 of the focal length of the objective, then the distance of the stadia rod *from the objective* (rigidly from E_2) is a hundred times the intercepted space on the rod, plus the focal length of the objective.

Again: if a base be measured in front of the instrument, *with its initial point a distance* f *in front of the object glass of the telescope,* then the rod may be held at any point on this base line, and *its distance from the initial point, and the space intercepted by the extreme wires, will be in constant ratio.*

The lines A_2F' and B_2F' in Fig. 1 show this relation, for they are the lines defining the space on the rod which is inter-

cepted by the extreme wires as the rod moves back and forth. Evidently the rod cannot approach so near as F', for then the image would be at an infinite distance behind the lens. Usually the extreme position of reticule does not correspond to a position of rod nearer than ten to fifteen feet.

3. It must be remembered that any motion of the eye-piece, with reference to the image and wires, is only made to accommodate different eyes, and has no effect in changing the relation of wire interval and image. The eye-piece is simply a magnifier with which to view the image and wires, but in all erecting-instruments it also re-inverts the image so as to make it appear upright. The effect of the eye-piece has no place in the discussion of stadia formula.

4. If the distance of the stadia is to be reckoned from the centre of the instrument, which it usually is, and if this distance $= d$, and the distance from the centre of the instrument to the objective (CE_2 in Fig. 1) $= c$, then we have, from (3),

$$d = f_2 + c = \frac{f}{i}s + f + c. \qquad (4)$$

Since f, i, and c are constant for any instrument, we may measure f and c directly, and then find the value of i by a single observation. Proceed as follows: —

1°. Measure the distance from the centre of the instrument (intersection of plumb line with telescope) to the objective, and call this c.

2°. Focus the instrument on a distant point, preferably the moon or a star, and measure the distance from the plane of the cross wires to the objective, and call this f.

3°. Set up the instrument, and measure the distance $f + c$ forward from the plumb line, and set a mark. From this mark as an initial point, measure off any convenient base, as 400 feet.

4°. Hold the rod at the end of this base, and measure the space intercepted by the extreme wires. If we call the length

of this base b, and the distance intercepted s, then we have, from equation (3),

$$b = \frac{f}{i}s$$

or

$$i = \frac{s}{b}f. \tag{5}$$

Here we have the value of i in terms of known quantities.

If it is desirable to set the wires at such a distance apart that $\frac{s}{b}$ will be a given ratio, as $\frac{1}{100}$, then i must equal $0.01f$. It is possible to set the wires by this means to any scale, so that a rod of given length may read any desired maximum distance.

5. If it is desired that $\frac{f}{i}$ should be determined with great accuracy for a given instrument, with wires already set, so as to have a co-efficient of reduction for distance, for readings on a rod graduated to feet and tenths, for instance, proceed as follows: —

Make two sets of observations for distance and intercepted interval. The distances should differ widely, as 50 feet and 500 feet, or 100 feet and 1000 feet, according to the length of rod used. The shorter distance should not be less than 50 feet, and the longer one not more than 1,000 feet with the most favorable conditions of the atmosphere. The distances are to be measured from the centre of the instrument. Make several careful determinations of the wire interval at each position of the rod, and take the mean of all the results at each distance, and call that the wire interval, s, for that distance, d. We then have two equations and two unknown quantities, these latter being $\frac{f}{i}$ and $(f + c)$ in the formula, equation (4),

$$d = \frac{f}{i}s + (f + c).$$

Here the d and s are observed, and $\frac{f}{i}$ and $(f + c)$ are found. Knowing these, a table could be prepared giving values of d for any tabular value of s for that instrument.

This applies to the reading of distances from levelling-rods.

Some engineers prefer, in this case, to observe the wire interval for various measured distances, from the shortest to the longest, to be read in practice, and prepare a table by interpolation. If the observed positions are sufficiently numerous, this method should give identical results with those obtained by the use of the formula. The two methods may be used to check each other.

6. From equation (4) we see that the distance of the rod from the centre of the instrument is a constant ratio $\left(\frac{f}{i}\right)$ times the intercepted space on the rod, plus a constant $(f + c)$.

If diagrams or designs be drawn on the stadia rod to the scale $\frac{i}{f}$, or so that $10 \times \frac{i}{f}$ yards on the rod would correspond to 10 yards in distance, and if the rod were decorated with symbols of this size, then the distance of the rod from the instrument could be read at once by noting how many symbols were intercepted between the wires. To this distance must then be added the small distance $(f + c)$, which is from 10 to 16 inches in ordinary field transits. On all side readings, taken only to locate points on a map, this correction need not be added, as one foot is far within the possibilities of plotting.

7. On the government surveys, the base is usually measured *from the centre of the instrument;* and its length is taken as about a mean of those which the stadia is intended to measure, and the symbols scaled by this reading. Then, of course, the distance read is always in error by a small amount, except when it is the same as the base for which it was graduated. For all shorter distances the reading is too small, and for all greater distances the reading is too large. Sometimes several different lengths of base are taken, as 400, 600, and 800 feet,

all from centre of instrument, and a mean value of wire interval used for giving the scale for the diagrams. This is practically the same as the other, for in either case the scale is correct for but a single distance.

8. The correction to any reading on a stadia so graduated, in order to give the distance from the centre of the instrument, is

$$K = (c + f)\left(1 - \frac{B}{B'}\right), \tag{6}$$

where K = correction, in feet,
B = distance read on stadia, in feet,
B' = length of base, in feet, for which the stadia was graduated.

If $B' = 1000$ feet, $B = 100$ feet, and $c + f = 1.5$ feet, then

$$K = 1.5\left(1 - \frac{100}{1000}\right) = +1.35 \text{ feet.}$$

If B had been 2000 feet, then

$$K = 1.5\left(1 - \frac{2000}{1000}\right) = -1.5 \text{ feet.}$$

These corrections are not usually applied.

9. Another method of determining the scale for graduating the rod is, to measure the base from the plumb line, as above, and then, from a fixed point on the lower part of the rod, find the intervals that correspond to various distances, as 100 feet, 200 feet, 300 feet, etc., and mark these on the board, always keeping the lower wire on the fixed, initial point of the rod. Then each 100-foot space is subdivided into ten equal parts, or symbols; so that, in reading the rod afterwards, if the lower wire is always set on the initial point, the reading always gives the correct distance from the centre of the instrument.

The objection to this method is, that the initial point on the rod cannot always be seen, on account of obstructions.

II. *Adaptation of Formulæ to Inclined Sights.*

10. The previous discussion is applicable to horizontal sights only.

If the rod be held on the top of a hill, and the telescope pointed towards it, the reading on the rod will give the linear distance from instrument to rod, ***provided the rod be held perpendicular to the line of sight.*** As it would be inconvenient to do this, let the rod be held vertical in all cases. When the line of sight is inclined to the rod, the space intercepted is increased in the ratio of 1 to the cos of the angle with the horizon.

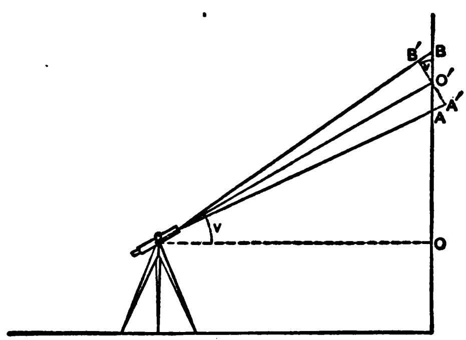

FIG. 2.

Thus, the space $A'B'$ (Fig. 2) for the rod perpendicular to the line of sight becomes AB for the rod vertical. But $A'B' = AB \cos v$.*

Let $A'B' = r'$, the reading on the stadia for perpendicular position; and

Let $AB = r$, the actual reading obtained for a vertical position.

Then $r' = r \cos v$.

But in equation (4) we have $\frac{f}{i'}s = r'$, and therefore $r' + c$

* This assumes that $A'B'$ is perpendicular to CB and CA, which it is practically, since the angle ACO' is so very small, usually about 15'.

$+f$ is the distance CO'; whereas, the distance on the horizontal, CO, is generally desired: and for this we have

$$CO = d = CO' \cos v = (r' + c + f) \cos v$$
$$= r \cos^2 v + (c + f) \cos v. \qquad (7)$$

This is the equation for reducing all readings on the stadia to the corresponding horizontal distances.

The vertical distance of O' above O is equal to CO' sin V.

But

$$CO' = r' + f + c = r \cos v + f + c,$$

hence

$$OO' = h = r \cos v \sin v + (f + c) \sin v$$
$$= \tfrac{1}{2} r \sin 2 v + (f + c) \sin v. \qquad (8)$$

Equation (8) is used for finding the elevation of the point on which the stadia is held above or below the instrument station.

11. Table I. gives the values d and h computed from these formulæ for a stadia reading of 100 feet (or metres, or yards), with varying angles up to 30°.

It will be noted that the second term in the right member of equations (7) and (8) is always small, and its value depends on the instrument used. The values of this term are taken out separately in the table: and three sets of values are given of $(c + f)$; viz., 0.75 feet, 1.00 feet, and 1.25 feet. If the work does not require great accuracy, these small corrections may be omitted.

The use of the table directly involves a multiplication for every result obtained. Thus, if the stadia reads 460 feet, the angle of inclination 6° 20′, and we have $f + c = 1$ foot, then

$$d = 4.60 \times 98.78 + 0.99 = 455.4 \text{ feet},$$

and

$$h = 4.60 \times 10.96 + 0.11 = 50.53 \text{ feet}.$$

12. The table is not generally used for reductions for d when the angle of elevation is less than 3 to 5 degrees. When $v = 5° 44'$, this reduction amounts to just *one per cent.* When an error of 1 in 100 can be allowed, then the reduction to the horizontal would not be used under 6°. If the second term in $c + f$ be also neglected, these two errors tend to compensate; and if $c + f$ for the instrument used is 1 foot, and both these corrections be omitted, they do exactly compensate when the

stadia	reading	is	100	feet,	vertical	angle	5° 44'.
"	"	"	200	"	"	"	4° 04'.
"	"	"	300	"	"	"	3° 20'.
"	"	"	400	"	"	"	2° 52'.
"	"	"	500	"	"	"	2° 32'.
"	"	"	1000	"	"	"	1° 46'.
"	"	"	2000	"	"	"	1° 18'.

Therefore the reduction to the horizontal need *never* be made when v is less than 2°, and it generally may be neglected when v is less than 6°.

In obtaining the difference of elevation, h, the term in $c + f$ may be omitted for all angles under 6° if errors of 0.1 foot are not important. For elevations on the main line, however, this term should always be included.

In practice, therefore, the tables are mostly used to obtain the difference of elevation from the given stadia reading and angle of elevation.

13. Since the use of these tables involves a multiplication each time, and since a table for varying distances and angles would be very voluminous, it is preferable to take out the elevations from a diagram. Such a diagram has been prepared, to be used in place of the table. It is arranged with both co-ordinates in feet, but can be used for both co-ordinates in metres, since the same unit is used for both. It will only be necessary to renumber the divisions, to adapt it to the new scale.

This diagram has been prepared with great care, and is arranged to give distances to 500 yards or metres, or 1,500 feet, with elevations to 50 feet. For longer distances or higher elevations for a single pointing, the results may be obtained from the table. Elevations are taken off from the diagram, to the nearest tenth of a foot, with great readiness; as the smallest spaces are 2 millimetres square, and these correspond to two-tenths of a foot in elevation. It is of more convenient use than extended tables, and is just as accurate; the nearest tenth of a foot being quite as exact as one is warranted in writing elevations when obtained in this manner.

Corrections to the distances read are also obtained from this diagram for large vertical angles.*

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* The diagram is printed on heavy lithographic paper 20 by 24 inches, from an engraved plate, and can be had from the publishers of this volume. Price 50 cents, post-paid.

CHAPTER II.

THE INSTRUMENTS.

1. *The Transit.*—That the transit may be best adapted to this work, there are certain features it should possess, though all of them are by no means essential. They will be named in the order of their importance.

1°. The horizontal limb should be graduated from zero to 360°, preferably in the direction of the movement of the hands of a watch.

2°. The instrument should have a vertical circle rigidly attached to the telescope axis, and not simply an arm that is fastened by a clamp screw, and which reads on a fixed arc below. So much depends on the vertical circle holding its adjustment, that its arrangement should be the best possible. Since the telescope is not transited, the vertical circle need not be complete.

3°. The telescope should be inverting, for two reasons: *first*, in order to dispense with two of the lenses, and so obtain a better definition of image; and *second*, that the objective may have a longer focal length, thus giving a flatter image and a less distorted field.

4°. The stadia wires should be fixed instead of adjustable, as in the latter case they are not stable enough to be reliable.

5°. The bubbles on the plate of the instrument should be rather delicate, so that a slight change in level may become apparent. They should also hold their adjustments well. This is very important, in order that the readings of the vertical angles may be reliable. It is also of great importance in

carrying azimuth where the stations are not on the same level.

6°. The horizontal circle should read to thirty seconds; and there should be no eccentricity, so that one vernier reading shall be practically as good as two.

7°. The instrument (or tripod) should have an adjustable centre, for convenience of setting over points.

8°. A solar attachment to the telescope will be found very convenient. In most regions the azimuth can be checked up by the reading of the needle, but in many places this is not reliable. In such cases, the solar attachment made by Fauth & Co. of Washington will be found most satisfactory. It can be put upon any transit, will revolve with the telescope without interfering, screws on and off, and is exceedingly simple and accurate in its workings, the reading on the sun being taken through a telescope, and so a very accurate setting made. It is also cheaper than others, the cost being only about forty dollars. With this attachment, a solar observation gives the true meridian, without any computation whatever, when the sun's declination is known. The attachment itself has no graduated circles or arcs, the vertical circle of the transit alone being used.

2. *Setting the Cross Wires.* — It is best to obtain fresh wires by having a spider spin them when needed. A fresh wire is better than an old one, because it is more elastic. An old wire may be made quite elastic, however, by steaming it a few seconds. To provide for emergencies, it is well to procure a cocoon of good wires, and carry it with you in the field. Some wires seem flat and twisted, like an augur shank. The wires for stadia work should be small, round, and opaque. A small black woods spider makes a very good web for stadia work. The cocoon of such a spider is a valuable addition to an instrumental outfit. A pair of dividers, and a small bottle of shellac dissolved in alcohol to the consistency of honey, are all that is needed for setting the wires.

Scratches must be made across the face of the reticule

where the wires are to lie. These must be made with great care, so as to have them equally spaced from the middle wire, parallel to each other, and perpendicular to the vertical wire. The distance apart of the extreme wires is to be computed by equation (5) for any desired scale on the rod.

Take a piece of web on the points of a pair of dividers, by wrapping the ends several times about the points, which should be separated by about an inch; stretch the wire, by spreading the dividers, as much as it will bear; and lay the dividers across the reticule in such a way that the web comes in place. The dividers must be supported underneath, so that the points will drop just a trifle below the top of the reticule: otherwise they would break the web. Move the dividers until the web is seen, by the aid of a magnifying-glass (the eye-piece will do), to be in exact position. Then take a little shellac on the end of a small stick or brush, and touch the reticule over the web, being careful to have no lateral motion in the movement. The shellac will harden in a few minutes, when the dividers may be removed. Shellac is not soluble in water.

3. *Graduating the Stadia Rod.*—The stadia rod is usually a board one inch thick, four or five inches wide, and twelve to fourteen feet long. Sometimes this is stiffened by a piece on the back. To graduate the rod, it is necessary to know what space on the rod corresponds to a hundred feet (or yards, or metres) in distance. Either of the three methods cited on pp. 7–8 may be used for doing this, but the first is recommended. Thus, measure off $c + f$ in front of the plumb line, and set a point. From this point measure off any convenient base, as 200 yards, on level ground, and hold the blank rod (which has had at least two coats of white paint) at the end of this base line. Have a fixed mark or target on the upper part of the rod, on which the upper wire is set. Have an assistant record the position of the lower wire as he is directed by the observer. Some sort of an open target is good for this purpose, but any scheme is sufficient that will enable the observer to fix the position of the extreme wires at the same moment with

exactness. This work should be done when there is no wind, and when the atmosphere is very steady: a calm, cloudy day is best. Repeat the operation until the number of results, or their accordance, shows that the mean will give a good result. If the base was 200 yards long, divide this space into two equal parts, then each of these parts into ten smaller parts, and finally each small space into five equal parts; and one of these last divisions represents two yards in distance. Diagrams are then to be constructed on this scale, in such a way that the number of symbols can be readily estimated at the greatest distance at which the rod is to be read. The individual symbols should be at least three inches across; so that, if one of these is to represent *ten units*, as yards or metres, then 100 units will cover 2½ feet, and a rod 14 feet long will read a distance of 560 units (yards or metres). If it is desired to read distances of a quarter of a mile or more, the rod should be graduated to read to yards (or five-foot units, or metres); but, if it is not to be used for distances over 500 to 1,000 feet, it might be graduated to read to feet. This question must be decided before the wires are set, and then they must be spaced accordingly.

In measuring the base, care should be taken to test the chain or tape carefully by some standard.

If the rod is to be graduated to read to feet, of course the base should be some even hundreds of feet, as 600.

4. In Fig. 3 are shown three designs for stadia rods which have been in long use, and are found to work well. They are intended to be all in black on a white ground.* It will be noticed that the shortest lines in these diagrams all cover a space of *two units* on the rod. In diagrams 2 and 3 the units are either yards or metres, while in 1 they are units of five feet each. The successive units are found at the middles and limits of these lines and spaces. Wherever the wire falls, there should be a *white ground* on some part of the cross section; and the

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* Some engineers prefer red on the 100-unit figures.

more white ground the better, provided the figures are distinct. The black paint may be put on heavy, so that one coat will be sufficient.

The 50 and 100 unit marks should be distinguished by special designs. There should usually be at least two boards with each instrument, and sometimes three and four are needed. Of course, these are all duplicates. After the *unit scale* is obtained, or the space on the rod corresponding to a hundred units in distance, these 100-unit spaces should be so distributed as to be *symmetrical with reference to the ends of the rod.* The reason of this will appear later. Having determined how many 100-unit spaces there will be on the rod, fix the position of the two end 100-unit symbols with reference to this symmetry, and then the rod is subdivided from these points.

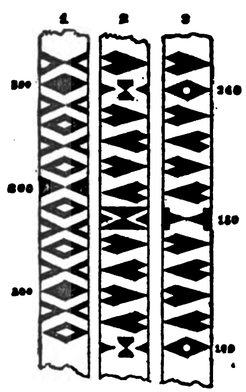

FIG. 3.

Special pains should be taken to have the angular points of the diagrams well defined and in position. These points are on the lines of subdivision of the rod.

After one rod is subdivided, the others of that set may be laid alongside, and all fastened rigidly together; and then, by means of a try square or T-square, the remaining rods may be marked off.

5. The wire interval should be tested every few months by

remeasuring a base, as was done for graduation, and reading the rod on it, to see if this shows the true measured distance. This is to provide against a possible change in the value of the wire interval. If the wires are stretched reasonably tight when they are put in, they seldom change. If they are too loose, they swell in wet weather, and may sag some. The reticule should be so firm that the variable strain on the adjusting-screws will not distort it appreciably.

If the wire interval is found to have changed, either the rods must be regraduated, or else a correction must be made to all readings of importance. What are called the "side shots," which make up a large proportion of the readings taken, would not need to be corrected.

If the wires are adjustable, any unit scale may be chosen at pleasure, and the wires adjusted to this scale. Then, if the intervals change, the matter is corrected by adjusting the wires. The adjustable wires are generally used to obtain distances from levelling-rods, where it is desirable that each foot on the rod shall correspond to a hundred feet in distance.

For the ordinary stadia rods, fixed wires are preferable.

CHAPTER III.

GENERAL TOPOGRAPHICAL SURVEYING.

I. *Field Work.*

1. LET it be required to make a topographical survey of either a small tract, a continuous shore line, or of a large area, for the purpose of making a contour map.

In case of the small tract, any point may be taken as a point of reference, and the survey referred to it as an origin. In case of an extended region, a series of points should be determined with reference to each other, both in geographical position and in elevation. These determined points should not be more than about three miles apart. The points of elevation or bench marks need not be identical with those fixed in geographical position. These last are best determined by a system of triangulation, and are called "triangulation stations." In the succeeding discussion, the symbol △ will be used for *triangulation station*, and B.M. for *bench mark*.

First, a system of triangulation points is established, the angles observed, azimuths and distances computed, and the stations plotted to scale on the sheet which is to contain the map. This plotting is best done, for small areas, by computing the rectangular co-ordinates (latitudes and departures), and plotting them from fixed lines which have been drawn upon the map, accurately dividing it into squares of 1,000 or 5,000 units on a side. They may, however, be plotted directly from the polar co-ordinates (azimuth and distance) as given by the triangulation reduction. For this purpose, the sheet on which the map is first drawn, called the *field sheet*, should have

a protractor circle printed upon it, about twelve inches in diameter. These *protractor sheets* of drawing-paper can be obtained of most dealers in drawing-materials, or the protractor circle may be printed to order on any given size or quality of paper.* These protractor circles are very accurate, and are graduated to 15′ of arc. Plotting can be done to about the nearest 5′.

Second, a line of levels is run, leaving B.M.'s at convenient points whose elevations are computed, all referred to a common datum. If the △'s are not also B.M.'s, then a B.M. should be left in the near vicinity of each △. This is not essential, however.

Third, the topographical survey is then made, and referred to, or hung upon, this skeleton system of △'s and B.M.'s.

The topographical party should consist of the observer, a recorder, two or three stadia men, and as many axemen as may be necessary, generally not more than two.

The azimuth, preferably referred to the true meridian, is known for every line joining two △'s, as well as the length of such line.

2. Set up the transit over a △, and set the horizontal circle (which should be graduated continuously from 0° to 360° in the direction of the hands of a watch) so that vernier A will read the same as the azimuth of the triangulation line by which the instrument is to be oriented. Clamp the plates in this position, and set the telescope to read on the distant △. Now clamp the instrument below, so as to fix the horizontal limb, and unclamp above. The azimuths of the triangulation lines are generally referred to the south point as the zero, and in small systems of this sort the forward and back azimuths are taken to be 180° apart. When the instrument has been set and clamped, all subsequent readings taken at that station are given in azimuth by the readings of vernier A on the horizontal limb. For any pointing, therefore, the reading of this

* Messrs. Queen & Co., Philadelphia, or Blattner & Adam of St. Louis, can furnish such sheets.

vernier gives the azimuth of the point referred to the true meridian, and the rod reading gives the distance of the point from the instrument station. These enable the point to be plotted on the map. To draw the contour lines, elevations must also be known.

If the elevation of the △ is known, measure the height of instrument (centre of telescope) above the △, on the stadia, as soon as the instrument is levelled up over that station. Suppose this comes to the 212-unit mark. Write in the note-book, as a part of the general heading for that station, "Ht. of Inst. = 212." Then, for all readings from that station for elevations, bring the middle horizontal wire to the 212-unit mark on the rod, and read the vertical angle. From this inclination and distance, the height of the point above or below the instrument station is found. If the rod be graduated symmetrically with reference to the two ends, one need not be careful always to keep the same end down, and so errors from this cause are avoided.

3. The record in the note-book consists of —

1°. *A Description of the Point,* as, "N.E. cor. of house," "intersec. of roads," "top of bank," "C.P." for "contour point," which is taken only to assist in drawing the contours, "⊡ 16" for "stadia station 16," etc.

2°. *Reading of Ver. A.*

3°. *Distance.*

4°. *Vert. Angle.*

These four columns are all that are used in the field. There should be two additional columns on the left-hand page, for reductions; viz., —

5°. *Difference of Elevation,* corresponding to the given vertical angle and distance, and which is taken from a table or diagram.

6°. *Elevation,* which is the true elevation of each point referred to the common datum.

The right-hand page should be reserved for sketching.

It will be found most convenient to let the sketching pro-

ceed from the bottom to the top of the page; as in this case the recorder can have his book properly oriented as he holds it open before him, and looks forward along the line. The notes may advance from top to bottom or *vice versa*, as desired. If there are many "side shots" from each instrument station, one page will not usually contain the notes for more than two stations, and sometimes not even for one.

The sketch is simply to aid the engineer when he comes to plot the work, and may often be omitted altogether. One soon becomes accustomed to impressing the characteristics of a landscape on his memory so as to be able to interpret his notes almost as well as though he had made elaborate sketches. For beginners the sketches should be made with care. The observer should usually make his own sketches and plot his own work.

4. After the instrument is oriented over a station, and its height taken on the stadia, the stadia men go about holding the rods at all points which are to be plotted on the map, either in position or in elevation, or both. The choice of points depends altogether on the character of the survey; but, since a single holding of the rod gives the three co-ordinates of any point within a radius of a quarter of a mile, it is evident the method is complete, and that all necessary information can thus be obtained. For very long sights, the partial wire intervals (intervals between an extreme and the middle wire) may be read separately on the stadia, and in this way twice as great a distance read as the rod was designed for. The limit of good reading is, however, usually determined by the state of the atmosphere, rather than by the length of the rod. When the air is very tremulous, good readings cannot be made over distances greater than 500 feet; while, when the atmosphere is very steady, a half-mile may be read with equal facility.

5. Before the instrument is removed from the first station, the forward stadia man selects a suitable site for the next instrument station (generally called *stadia station*, and marked ⊡, to distinguish it from a triangulation station, △), and drives

a peg or hub at this point. This peg is to be marked in red chalk, with its proper number, and should have a taller marking-stake driven by the side of it. The peg for the ⊡ should be large enough to be stable; for it must serve as a reference point, both in position and elevation, during the period of the survey. It is often desirable to start a branch line, or to duplicate some portion of the work, with one of these stations as the starting-point; and, since each ⊡ is determined, in position and elevation, with reference to all the others, one can start a branch line from one of these as readily as from a △. It is not usually necessary to put a tack in the top, but the centre may be taken as the point of reference. The stadia man first holds his stadia carefully over the centre of this ⊡, *with its edge towards the instrument*, so as to enable the observer to get a more accurate setting for azimuth. The observer could just as well bisect the face of the rod; but, if held in this position, the centre of the rod may not be so nearly over the centre of the peg as when held edgewise. This holding of the rod edgewise for azimuth checks the carelessness of the stadia man, and is only done with readings on instrument stations.

At a signal from the observer, the stadia is turned with its face to the instrument, and the observer reads the distance and vertical angle.

It is advisable, in good work, to re-orient and relevel the instrument just before reading to the forward ⊡. The transit is very apt to get out of level after being used for some time, with more or less stepping around it, and the limb may have shifted slightly on the axis, both of which might be so slight as to make no material difference for the side readings, but which would be important in the continued line itself. It is best, therefore, to level up again, and reset on the back station, before reading to the forward one. If it is inconvenient for the rear rodman to go back to this station to give a reading, a visible mark should be left there, to enable the observer to reset upon it for azimuth, as it is not necessary to read distance and vertical angle again.

When the instrument is moved, it is set up over the new station, and the new height of instrument determined and recorded. The rear stadia man is now holding his rod, edgewise, on the station just left; and by this the observer orients his instrument, *making vernier A read 180° different from its previous reading on this line.* Clamping the plates at this reading, the telescope is turned upon the rod on the back station, and the lower plate clamped for this position. The circle is now oriented, so that, for a zero reading of vernier H, the telescope points south.

It will be noted that the telescope is never transited in this work.

The *distance* and *vertical angle* should both be reread, on this back reading, for a check. If the vertical circle is not in exact adjustment, this second reading of the vertical angle will show it, for the numerical value of the angle should be the same, with the opposite sign. If they are not the same, then the numerical mean of the two is the true angle of elevation, and the difference between this and the real readings is the index error of the vertical circle. This error may be corrected in the reduction, or the vernier on the vertical circle may be adjusted.

The second reading of the vertical angle on the stadia stakes is thus seen to furnish a constant check on the adjustment of the vertical circle, and should therefore never be neglected. If the circle is out of adjustment by a small amount, as one minute or less, in ordinary work it would not be necessary either to adjust it or to correct the readings on side shots, for the elevation of contour points is not required with such extreme accuracy. The mean of the two readings on stadia stakes would still give the true difference of elevation between them, so that there would be no continued error in the work.

6. The work proceeds in this manner until the next △ is reached. In coming to this station, it is treated exactly as though it were a new ⊡; and the forward reading to it, and the

back reading from it, are identical with those of any two consecutive ⊡'s. Having thus occupied the second △, and having oriented the instrument by the last ⊡, turn the telescope upon some other △ whose azimuth from this one is known. The reading of vernier A for this pointing should be this azimuth, and the difference between this reading and the known azimuth of the line is the accumulated error in azimuth due to carrying it over the stadia line. This error should not exceed five minutes in the course of two or three miles in good work.

The check in distance is to be found from plotting the line, or from computing the co-ordinates of the single triangulation line, and also of the meandered line, and comparing the results.

The elevations are checked by computing the elevation of the new △ from the stadia line, and comparing this with the known elevation from the line of levels.

In case the elevations of the △'s are not given, but only certain B.M.'s in their vicinity, then the check can be made on these just the same. Thus, in starting, read the stadia on the neighboring B.M., and from this vertical angle compute the elevation of the △ over which the instrument sets, and then proceed as before. In a similar manner, the check for elevation at the end of the line may be made on a B.M. as well as on the △.

A quick observer will keep two or three stadia men busy giving him points; so that in flat, open country, with long sights, it may be advisable to have three or even four stadia men for each instrument. In hilly country more time will be required in making the sketches, and hence fewer stadia men are required.

7. After the instrument is oriented at each new station, the needle should be read as a check. To make this needle reading agree with the readings of the verniers on the horizontal circle (the north end with vernier A, and the south end with vernier B, for instance), graduate an annular paper disk the size of the needle circle, and figure it continuously from 0° to 360°, *in the reverse direction to that on the horizontal limb of the instru-*

ment, and paste it on the graduated needle circle in such a position that the north end of the needle reads zero when the telescope is pointing south. If the variation is 6° east, this will bring the zero of the paper scale 6° east of south on the needle circle. This position of the paper circle is then good within the region of this variation of the needle. When the survey extends into a region where the variation is different, the scale will have to be reset.

With these conditions, when the instrument is oriented for a zero reading when the telescope is south, the reading of the north end of the needle will always agree with the reading of vernier A, and the south end with vernier B. It is so easy a matter to let the needle down, and examine at each ⊡ to see if this be so, that it well pays the trouble. No record need be made of this reading, as it is only used to check large errors.

II. *Reducing the Notes.*

8. The only reduction necessary on the notes is, to find the elevation of all the points taken, with reference to the fixed datum, and sometimes to correct the distance read on the rod for inclined sights. The difference of elevation between the ⊡ and any point read to, as well as the correction to the horizontal distance, can be taken from Table I. or from the diagram. The methods of using these have been explained (see pp. 11–13). After the differences of elevation are taken out, the final elevations of the points are to be computed by adding algebraically the difference of elevation to the elevation of the ⊡.

The following is a sample page with these reductions:—

GAZZAM, *Observer.*

APRIL 20, 1883. BAIER, *Recorder.*

At ⊡ 4. Ht. of Inst. = 87. Elevation = 24′.94.

Object.	Azimuth. Ver. A.	Distance.	Vert. Angle.	Difference of Elevation.	Elevation above Datum.
		yds.			
⊡ 3	328° 10′	199	−0° 10′	− 1′.56	–
Bridge	127° 40′	70	+0° 32′	+ 1′.9	26′.8
S.E. cor. of house	142° 35′	90	+0° 15′	+ 1′.2	26′.1
On road	180° 25′	114	+0° 7′	+ 0′.7	25′.6
Water level, foot of hill .	230° 15′	224	−0° 57′	−10′.9	14′.0
⊡ 5	128° 33′ 30″	216	+0° 55′	+10′.38	–
C. P.	190° 48′	210	+1° 2′	+11′.4	36′.3
At ⊡ 5.	**Ht. of Inst. = 78.**		**Mean =**	**+10′.26.**	**35′.20.**
⊡ 4	308° 33′ 30″	215	−0° 54′	−10′.13	–
S.W. cor. of house . . .	43° 30′	104	+3° 3′	+16′.0	51′.2
Edge of bank	332° 10′	98	+1° 57′	+10′.1	45′.3
S.E. cor. of R.R. station .	85° 30′	158	+1° 2′	+ 8′.5	43′.7
Railroad track	43° 55′	40	+2° 53′	+ 6′.0	41′.2
" "	79° 30′	270	+0° 9′	+ 2′.1	37′.3
⊡ 6	79° 30′	200	−0° 2′	− 0′.36	–
At ⊡ 6.	**Ht. of Inst. = 79.**		**Mean =**	**−0′.54.**	**34′.66.**
⊡ 5	259° 30′	200	+0° 4′	+ 0′.72	–
Cor. of house	277° 55′	112	+3° 26′	+19′.7	54′.4
Top of hill	87° 25′	198	+4° 48′	+49′.3	84′.0
Wagon road	58° 15′	186	+4° 25′	+42′.9	77′.6
⊡ 8	40° 37′	216—3 213	+6° 33′	+73′.53	–
C. P.	41° 45′	111	+4° 41′	+27′.0	61′.7
⊡ 7	5° 25′	194	+0° 12′	+ 2′.04	–

It will be noted that the reading on ⊡ 5 from ⊡ 4 has a distance of 216 yards, and a vertical angle of $+ 0° 55'$; while on the back reading, from ⊡ 5 to ⊡ 4 the distance is 215 yards, and the vertical angle $- 0° 54'$. The distance was probably between 215 and 216 yards, and the vertical circle was probably slightly out of adjustment. The difference of elevation is taken out for both cases, however, being respectively 10.38 feet and 10.13 feet. The mean of these is 10.26 feet, which stands as a part of the general heading at ⊡ 5. The true elevation of ⊡ 5 is then found by adding 10.26 to 24.94, giving 35.20 feet, which is also set down as part of the general heading.

The elevations on the side readings from this station can now be taken out. These side elevations are only used for obtaining the contours, and hence are only taken out to tenths of a foot. When the contours are ten feet apart or more, these side elevations need only be taken out to the nearest foot. The elevations of the stadia stations should, however, always be taken out to hundredths, to prevent an accumulation of errors in the line.

The reduction for distance may also be taken from that portion of the diagram arranged for this purpose. This is used the same as the other portion; and the *correction* is found, which is to be always subtracted from the rod reading. Thus, in the reading on ⊡ 8 from ⊡ 6, we have a reading of 216 yards, and a vertical angle of $6° 33'$. The correction here is $216 \times 1.3 = 2.8$ yards, as found from the table. Calling this 3 yards, it is subtracted from the 216, leaving 213 yards as the distance to be plotted. It is only the stadia-line distances that need ever be corrected in this way, the corrections being usually so small that it is not important on the side shots.

It will be noted that two ⊡'s were set from ⊡ 6. This was done because a branch line was run from ⊡ 6 over the bluffs. In order to make it unnecessary to occupy ⊡ 6 again when the branch line came to be run, ⊡ 8 was set while ⊡ 6 was occupied in the main-line work. When the branch line

came to be run, the instrument was taken directly to ⊡ 8, and oriented on ⊡ 6 by the readings previously taken from ⊡ 6.

The right-hand page of the note-book, opposite the notes given above, is occupied with a sketch of the locality, with the ⊡'s marked on, the general direction of the contour lines, the railroad, stream, houses, etc.*

III. *Plotting the Work.*

9. It is customary to first plot the stadia stations alone, from one △ to the next, to find whether or not it checks within reasonable limits. This part of the work should be done with extreme care, so that, if it does not check, it cannot be attributed to the plotting. In case it does not check within the desired limit, then the line of investigation will be about as follows until the error is found: —

1°. Replot the stadia line.

2°. Recompute and replot the triangulation line.

3°. By examining the discrepancy on the plot, try and decide whether the error is in azimuth or distance, and, if possible, where such error occurred, and its amount.

4°. Examine the note-book carefully, and see if there is any evidence of error there.

5°. If there is a large probability that the error is of a certain character, and that it occurred at a certain place, take the instrument to that station, set it up, and redetermine the azimuths or distances which seem to be in error.

6°. If there is no high probability of any certain errors to be examined for in this way, then go back and run the line over, *taking readings on ⊡'s only.* If the elevations had been found to check, the vertical angles may be omitted on this duplicate line: and, on the other hand, if the plot came out all right, but the elevations could not be made to check, then a

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* These notes were taken from a field-book of a topographical survey of Crève Cœur Lake by the engineering students of Washington University.

duplicate line must be run to determine this alone; and in this case the vertical angles between ⊡'s are all that need be read. In cases of this kind, it will be found a great help to have the ⊡'s so well marked that they can be readily found.

With reasonable care in reading and in the handling of the instrument, it will never be necessary to duplicate a line entire, for all readings between ⊡'s are checked. The vertical angles and distances are checked by reading them forward and back over every stadia line; and the azimuth is checked by the needle readings, and also when the second △ is reached.

If, in the progress of the work, the readings on the back ⊡ for distance and vertical angle do not fairly agree with these quantities as read from the previous station, the recorder should note the fact: and the observer should then re-examine these readings; and, if found to be right, the first readings, taken from the other station, should be questioned, and the mean not taken in the reduction.

10. To enable the observer to locate large errors in azimuth or distance, or both, it is a good practice to take azimuth readings to a common object from a series of consecutive stations, if such be possible. If the plot does not close, go back and plot in these azimuths; and if there has been no error in azimuth or distance between ⊡'s, and no error in reading the azimuths for these pointings, then all these lines will meet in a common point on the plot. If all but one intermediate line meet at a point, then the error probably was in reading the azimuth of this pointing alone. If several of the first pointings intersect in a point, and the remaining pointings of the set taken to this object intersect in another point, then it is highly probable that the error was in reading the azimuth or distance of the line connecting these two sets of ⊡'s; and the relative position of the points of intersection will enable the observer to decide whether the error was in azimuth or distance, and about how much. If, in this way, the error be located, the instrument can be taken to this point, and the readings retaken.

11. For plotting the stadia lines, a parallel ruler (moving on rollers) is very desirable; otherwise, triangles must be used. The plotting is done by setting the parallel ruler or triangle on the proper azimuth as found from the protractor printed on the sheet, moving it parallel to itself to the station from which the point is to be plotted, and drawing a pencil line in the right direction. Then, with a triangular scale, — or, better, with a pair of dividers and a scale of equal parts, — lay off the correct distance on this line; and this gives the point.

If the instrument was oriented in the field for a zero reading for a south pointing, then the protractor on the sheet must have its south point marked zero, and increase around to 360° in the same direction in which the limb of the instrument increases, preferably in the direction of the movement of the hands of a watch.

Having plotted the stadia line, and made it check, the next step is to go back and plot in the side readings. For doing this, a much more rapid method may be used than that described above.

Divide the sheet into squares by horizontal and vertical lines spaced uniformly at from 1,000 to 5,000 units apart, according to scale. These lines are to be used for orienting the auxiliary protractor, and also to test the paper for stretch or shrinkage.

The side readings are now plotted by the aid of a paper protractor, such as is shown in Fig. 4. This is made from a regular field protractor sheet. The graduated circle printed on the sheet is used; and this is some 12 inches in diameter, and graduated to 15 minutes. The sheet is trimmed down to near the graduated circle, and the edges divided, as shown in the figure, to any convenient small scale. This sheet is to be laid upon the plot, with its centre, *C*, coinciding with the ⊡. It is oriented by bringing the corresponding spaces on opposite edges to coincide with any one of the spaced lines on the plot. This circle then has its position parallel to that of the protractor circle printed on the sheet, and an azimuth taken from the one will agree with an azimuth taken from the

other. When this auxiliary protractor has been so centred and oriented, let it be held in place by weights. Now the part *ADEB* folds back, on the line *AB*, into the position indicated by the dotted lines. The portion *DEF* is cut out entirely; so that, when the flap is turned back, the space *AFB* is left open. This space is to be large enough to include the longest side readings when plotted to scale; that is, the radius, *CF*, of the circle, to the scale of the drawing, must exceed the longest readings. We now have a protractor circle about the ⊡, with this station for its centre.

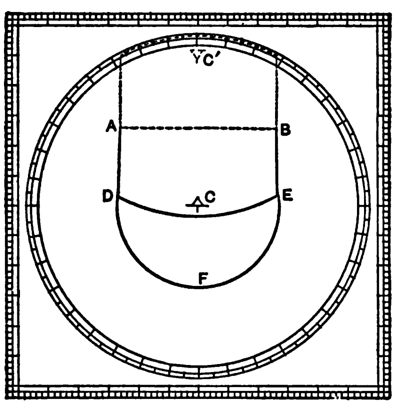

FIG. 4.

Take a triangular scale, select the side to be used in laying off the distances, and paste a piece of strong paper on the lower side at the zero point. Make a needle hole through this paper close to the edge, at the zero of the scale. Fasten a needle through this hole into the point which marks the exact position of the ⊡. The scale can now swing freely around the needle, on the auxiliary protractor; and its zero remains at the centre of the station from which the points are to be plotted.

To plot any point, swing the scale around to the proper azimuth, and at the proper distance mark with the pencil the position of the point. If this marks a feature of the landscape, it should be drawn in at once, before going farther; and

if the elevation of the point will be needed in sketching the contours, this should also be written in. For contour points, the elevation is all that is put down.

In this manner the points can be plotted very rapidly. A six-inch triangular scale, divided decimally, will be found best for this.

If there is very much of this work to be done, it might be found advisable to have a special scale constructed for the purpose. Fig. 5 is one form of such a scale drawn one-third

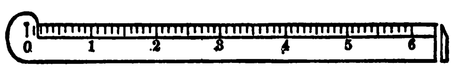

FIG. 5.

size, which would be found very convenient and cheap. It should be graduated on a bevel edge, and to such a scale that the units of distance used on the rod may be plotted to the scale of the drawing. The small needle hole, in line with the graduated edge, should be only large enough to fit the needle point used, so that there would be no play. The rule then turns on an accurate centre, which will not wear. Such scales, six inches long, could be constructed very cheaply of German silver by any instrument maker.

Other methods are employed for plotting the side shots, such as solid half-circle protractors, of paper or horn, weighted in position, with their centres over the station. This is oriented on a meridian drawn through the point, and then all the points plotted whose azimuth falls between 0° and 180°, when the protractor is laid over on the other side, and the remaining points plotted. In this case the ruler is laid across the protractor, with some even division at the station. This method is more trouble, less rapid, and defaces the drawing more, than the other method given above. The plotter should have an assistant to read off to him from the note-book. When all the elevations have been plotted, the contour lines are sketched in.

The above work is supposed to be done in pencil, and 6-H pencils alone should be used. It should keep pace with the field work as closely as possible, being done at night and at other times when the field work is prevented or delayed. In difficult ground, the plot could be carried into the field, and the contours sketched in on the ground. At least the stadia lines should be plotted up and checked before the observer leaves the immediate locality; and, where the elevations are checked on B.M.'s, these checks should also be immediately worked out. This much could be done each evening for that day's work.

IV. *The Final Map.*

12. If the map is to be worked up in ink or color, it may be copied upon another sheet of plain paper, without the printed protractor. Several of the field sheets would probably be drawn, to the same scale, upon a single, larger, plain sheet. The main features (contour lines, etc.) are traced off on tracing-paper, and this then oriented on the larger sheet by bringing certain fixed points to coincide, usually the △'s if there be any, and then the lines retraced with a dull point, leaving an indented mark under each line on the tracing. These are now drawn in ink, and the map completed by filling in with the appropriate topographical signs and representations. Plate I. shows a portion of such a finished map worked up in India ink. After the contour lines are determined, of course there is no further use for the elevations, so these do not appear. Each contour line should have its elevation, referred to the common datum, plainly marked.

The subject of projection of maps is considered in Chap. IX.

CHAPTER IV.

RAILROAD SURVEYING.

I. *Objects of the Survey.*

1. SINCE the transit and stadia are the best means of making a topographical survey, so they are the means that are best adapted to make a railroad survey so far as this is a topographical survey.

The map of a railroad survey may serve two purposes: —

First, to enable the engineer to make a better location of the line than could be done in the field.

Second, to give all necessary data relating to right of way, as the drawing of deeds, assessment of damages, etc.

In flat or gently undulating country, it may not be advisable to locate by a map; but even here the map is quite as essential for determining questions relating to the right of way.

In either case, therefore, a good topographical map of the line is of prime importance, and all the data for this map may be taken on the preliminary survey.*

Both these ends may be served by the same map. The method of location by contours (sometimes called "paper location") is often absolutely necessary in rough ground, but is still more often judicious in simpler work, inasmuch as a better location can often be made in this way.

* By "preliminary survey" is here meant a survey of a belt of country which it is expected will embrace the final line, and not a mere *reconnoissance* made to determine the feasibility of a line, or which of several lines is the best.

II. *The Field Work.*

2. In this case there would be no △'s or B.M.'s to check on; but the errors in distance and elevation would be no more, probably, than are now made on preliminary surveys. In fact, the errors in distance would not be nearly so great, unless the chain be tested frequently for length, and the greatest care taken on irregular ground. If a chain 100 feet long has 600 wearing-surfaces, which most of them have, and if each of these surfaces be supposed to wear 0.01 inch, which it will do in the course of a 200 or 300 mile survey, then the chain has lengthened by six inches, or the error in distance is now 1 in 200 from this cause alone. If we add to this the uncertain errors that come from chaining up and down hill, and over obstructed ground, it is certain that the stadia measures will be much the more accurate.

In the matter of elevations, since the local change of elevation is alone significant, and not the total difference of elevation of points at long distances apart, the line of levels carried by the stadia would be amply sufficient for a preliminary survey.

3. The following observations are applicable to the preliminary survey for final location, when it is expected the line will be included in the belt of country surveyed: —

1°. All data should be taken that will contribute to the solution of all questions of location, such as elevations for contour lines; streams requiring culverts, trestles, or bridges, and the necessary size of each, if possible; all depressions which cross the line, and will require a water-way, together with the approximate size of the area drained; highways and private roads or lanes; buildings of all kinds, fences, and hedges; character of surface, as rock, clay, sand, etc.; character of vegetation, as cultivated, forest, prairie, marsh, etc.; the location of any natural rock that may be used for structures on the line, such as culverts or abutments; high-water marks if in a bottom subject to overflow; and, in fact, all information which

will probably prove of value in determining the location, or in making up a report with estimates to the Board of directors, or in letting contracts for earthwork.

2°. All data that may be found useful in respect to land titles or right of way, or that may relate to claims for damages, such as section corners, boundaries, fences, buildings, streets, roads, lanes, farm roads, cultivated and uncultivated land, as well as such as may be cultivated, public and private grounds, orchards, forests, together with the value of the forest timber, mineral lands, stone quarries, proximity to villages, etc. Since the bearings and position of all boundary lines are of great importance in the matter of right of way, every such boundary should have at least two readings upon it in the field; and these should be as far apart as possible.

III. *The Maps.*

4. Before any plotting is done, two questions of importance must be decided. They are, *First,* whether one set of maps is to serve for both the location and for the further use of the company, or whether a set of contour maps, worked up in pencil, shall serve for the location, and another set for the continuous use of the company. *Second,* what shall be the scale of the maps? These will be argued separately.

5. *Whether one or two sets of maps* will be decided on, will depend largely on the care that is exercised with the locating-sheets. If these are carefully worked up for the location, and kept clean, they can be utilized for the final maps. If they become too badly soiled by field use, new sheets would probably be substituted for the uses of the company.

If it is expected, at the start, to have a different set of sheets for the final maps, then "protractor sheets" should be used for the location. In this case, plot on these sheets only such of the field notes as will contribute to the location; and these need only be plotted in pencil. When the location

has been made, such features may be transferred from the locating-sheets to the final maps as may be desired. These would consist mainly in the stadia stations, the contours, and the located line. The rest of the field notes may then be plotted on the final sheets, and the whole worked up in ink.

If, on the other hand, one set of maps is to serve both purposes, then it would, perhaps, be best to use plain sheets, as the protractor circle would somewhat disfigure the final maps. The protractor sheets would, however, furnish a ready means of taking off the bearings of lines from the final charts, which might be thought to compensate for the slight marring of the map's appearance. If plain sheets are chosen, then they should be divided into squares by lines drawn in ink parallel to the sides of the paper, in the direction of the cardinal points of the compass. Both the stadia stations and the side readings may then be plotted by means of the auxiliary protractor, this being oriented by the meridian lines on the sheet. Even here, only those readings would at first be plotted that will contribute to the location, and these marked in pencil. After the location has been decided on, and the location notes taken off, as described below, then the stadia stations, contour lines, the located line of road, and such other features as should be preserved on the final map, are inked in, and the map thoroughly cleaned. The rest of the field notes may now be plotted, and the map finished up.

If the road runs through a settled region, the questions of right of way are among the first things to be settled; so that preliminary maps showing the relation of the road belt to the property lines are essential to the settlement of damages, and to obtaining the right of way from the property holders. Coincident, therefore, with the making of maps to determine the location, must come the construction of preliminary right-of way maps or tracings. On these latter need be plotted only the boundary lines, fences, more important buildings, roads, etc., or just sufficient to enable the right-of-way agent to

negotiate intelligibly with the property owners.* Neither the locating nor the final map should be on a continuous roll. The roll requires more room for storage, is more apt to get dusty, and is much more inconvenient for reference. When sheets are used, the survey plot covers a more or less narrow belt across the map. One of the edges of the sheet, either where the plot enters upon it or disappears from it, should be trimmed straight, and the plot extended quite to this edge. This edge is then made to coincide with one of the parallel or meridian lines of the next sheet; so that, when the line is plotted, the sheets may be tacked down in such a way as to show the continuous plot of the survey.

6. *The scale of the map* will depend on whether or not separate sets of charts are to serve the purposes of location and of the continuous use of the company. For the purpose of location, a scale of 400 feet to one inch does very well; but for the final detail sheets, the scale should be larger. If both purposes are to be served by one set of maps, then the scale should be about 200 feet to one inch,† with 5 or 10 foot contours. The sheets should be about twenty by twenty-four inches.

IV. *Plotting the Survey.*

7. In case the map is plotted on a protractor sheet, the methods of plotting will be identical with those for general topographical work, except that here there will be no checks, either for distance, azimuth, or elevation, except such as are carried along or independently determined. For distance, there is no check, except the duplicate readings between instrument stations, unless the survey is through a region which has

* For an excellent article on the subject of right-of-way maps and permanent railway-property records, by Charles Paine, see The Railroad Gazette of Nov. 14, 1884.

† Some engineers prefer a scale of 100 feet to one inch for the final charts of the company.

already been surveyed. In this case the section lines may serve as a check on the distances.

The azimuth should be checked at every station by reading the needle, as described on p. 26, and also by independently determining the meridian frequently, either by a solar attachment or by a stellar observation. If the line is not nearly north and south, or, in other words, if it is extended materially in longitude, then the azimuth must be constantly corrected for convergence of meridians, as is shown in Chap. IX.

The elevations can only be checked by the duplicate readings between instrument stations.* All the greater care should be used, therefore, on readings between stations.

8. The first plotting, whether there are to be two sets of maps or one, will consist in representing on the sheet only such data as will assist in deciding on the location. These will be mainly contour points, streams, important buildings near the line, principal highways, other lines of railway, villages with their streets and alleys near the proposed location, the lines of demarkation between cultivated and timbered or wild land, etc. From the plotted elevations, aided by the sketches in the note-book, the contour lines are drawn in; and, if necessary, this may be done on the ground. This is sufficient for determining upon a location.

When this has been done, then the natural features, the contour lines, the stadia stations, and the located line, may be inked in (or transferred by means of tracing-paper, in case the final maps are to be on separate sheets), and the remainder of the notes plotted.

In drawing the contour lines in ink, make those upon barren or rocky land in black, and those on arable land in brown. If they are ten feet apart, make every tenth one very heavy, and every fifth one somewhat heavier than the others. If this be

* It may be observed, that the same lack of sufficient checks on the distance, azimuth, and elevation obtains with the ordinary preliminary survey with transit, level, and chain.

done, only the 50 and 100 foot contours need be numbered. In case a map does not contain at least two of these numbered contours, then every contour which does appear on the map should be numbered, giving its elevation above the datum of the survey.

The streams should be water-lined in blue, and an arrow should tell the direction of its flow. The name should also be given when possible.

All fences should be shown, and especial pains taken to represent division fences in their true position; for it is from this map that the deeds for the right of way are to be drawn.

Outhouses may be distinguished from dwellings by diagonal lines intersecting, and extending slightly beyond the outline. The character of the buildings may be shown by colors, as red for brick, yellow for frame, pale sepia for stone; the outlines always being in black.

The stadia stations should be left on the finished sheets; as, in case of a disputed boundary, or for other cause, the map may be replotted if the positions of the instrument stations are left on it. The numbers of the stations should, of course, be appended.

The magnetic bearings of boundary lines may be given on the map, or they may be determined, as occasion requires, by means of the auxiliary protractor and the true meridian lines when the variation of the needle is known. For this purpose, the magnetic meridian should be drawn on each map, diverging from one of the meridian lines, and the amount of the variation marked in degrees and minutes.

V. *Making the Location.*

9. When a preliminary survey is made, as above described, for the purpose of making what is called a "paper location," the location is first made on the map, and then staked out in the field.

Every railroad line is a combination of curves, tangents, and grades; and it is the proper combination of these which

makes a good location. If it be assumed that the line is to be included in the belt of country surveyed, then the map contains all the data necessary to enable the engineer to select the best arrangement of curves, tangents, and grades it is possible for him to obtain on this ground. This selection can be made with much more certainty than is possible on the ground, where the view is generally obstructed, and where grades are so deceptive.

It is no part of this treatise to discuss the various problems that enter into the question of a location, but only to show how to proceed to make a location that may satisfy any given set of conditions, by means of the contour map.

10. The contours themselves will enable the engineer to decide what the approximate grades will have to be. Suppose a grade of 0.5 foot in 100 feet, or 26.4 feet to the mile, has been fixed upon. It is now known that the line should follow the general course of the contours, except that it should cross a 10-foot contour every 2,000 feet. Spread the dividers to this distance, taken to scale, and mark off in a rough way these 2,000-foot distances as far as this grade is to extend; and do the same for the successive grades along the line. Knowing the grade of the line at the beginning of the sheet, the problem is to extend this line over the sheet so as to give the best location one can hope to get on this ground with the available means.

First, starting from the initial fixed point of line on the map, sketch in a line which will follow the contours exactly, crossing them, however, at such a rate as to give the necessary grade. This is the *cheapest* line, so far as cut and fill are concerned. Of course, where depressions or ridges are to be crossed, the line must cross over from a given contour on one side to the corresponding contour on the other, and then follow along the contour again.

Second, mark out a series of tangents and curves which will follow this sketched line as nearly as it is possible for a railroad to follow it. This will not be the final location, but it is

valuable for study. This line will be faulty from having too many and too sharp curves, and too little tangent.

Third, draw in a third line, as straight as possible, and with as low grade of curves as possible consistent with a reasonable amount of earthwork and a proper distribution of the same.

For the purpose of deciding what degree of curve is best suited to the ground for a given deflection angle, it is well to have a series of paper templets made, with the various curves for their outer and inner edges. Of course, these are cut with radii laid off to the scale of the drawing. It is still more convenient to have these curves, laid off to scale, on a piece of isinglass, horn, or tracing-paper (not linen), so that this can be laid upon the map, and the curve at once selected which will follow the contours most economically. Fig. 6 shows such a series of curves drawn to a scale of 1,600 feet to the inch.

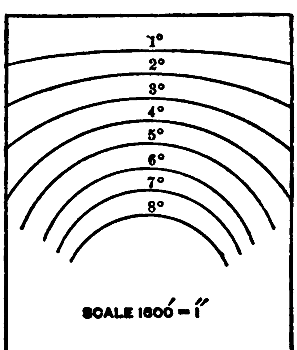

FIG. 6.

In this way the line is laid out over the map. The questions of greater or less curvature have been balanced against a less or greater first cost, and greater or less operating expense. The question of shifting it laterally has also been examined, and finally a definite location fixed upon which seems to answer best to the case in hand. When this is done, it only remains to make up the location notes from which the line is to be staked out.

11. The following is considered a good form for the location notes:—

Location Notes for A B C Railroad. From Map No.........

LINE.	Azimuth and Deflection Angles.	Length.	Station.	REMARKS.
		ft.		
T	260° 40′	1020	10 + 20	
3° C. R.	+18° 30′	617	16 + 37	
T	279° 10′	2670	43 + 7	
4° C. L.	−12° 20′	308	46 + 15	
T	266° 50′	680	52 + 95	

The first column designates the tangents and curves, and gives the degree of the curve, and the direction of its curvature, whether right or left. If it curve toward the right, the azimuth of the next tangent will be increased, and hence its sign is plus, and *vice versa.*

The second column gives the azimuths of the tangents and the deflection angles of the curves. Each azimuth is seen to be the algebraic sum of the two preceding angles.

The third column gives the lengths of the tangents as measured from the map, and the lengths of the curves as determined by dividing the deflection angle by the degree of the curve. Thus, $12° 20' = 12°.33$, and $12°.33 \div 4 = 308$, which is the length of the curve in feet.*

The fourth column gives the stations and pluses for the P.C.'s and the P.T.'s. These quantities are simply the continued sum of those in the third column.

* It is a great convenience to have at least one vernier, in railroad work, graduated to read to hundredths of a degree. The case here given is only one of many similar cases; but the principal advantage is in running the fractional parts of curves when the curve chosen is some even degree, as here taken.

The first, second, and fourth columns now give all the information necessary to stake out the line. The stadia is no longer to be used, but a transit and chain, as is ordinarily done.

The tangents need not be run out to their intersection; but when the P.C. is reached, according to the location notes taken from the map, set up the instrument, and stake out the curve as far as possible, or around to the P.T. In either case, when the instrument is to be moved, make a note of the forward azimuth, and go forward and orient on the last station the same as when moving between two ⊡'s. If the instrument be moved to the P.T. direct, then, after orienting back on the P.C., turn off to the azimuth given for the next tangent, and go ahead. The tangents could be run out to the intersection and the point occupied by the instrument, for a check, if thought desirable. *The telescope is never transited in laying out the line from the system of notes above given.*

With careful work, the line ought thus to be run out, and the curves put in at once. We have supposed there was no regular line cleared out on the preliminary, so the necessary clearing would all have to be done on the location.

A levelling party follows the transit, and obtains the data for constructing a profile and for determining the exact grades.

The stadia has served its purpose when it has enabled the engineer to select the most favorable position for the line. The transit, chain, and level must do the balance.* It is not improbable that occasional modifications will be introduced in the field, even though the survey and the location have been made with the greatest possible care.†

* For an article, with map, illustrating the advantage of having a good topographical survey, showing contours, in making a railroad location, see Location of the Northern Pacific Railroad across the Rocky Mountains, by J. C. Chesbrough, in Journal of the Association of Engineering Societies, vol. iii. p. 153.

† See editorials on the Use and Abuse of Topography in railroad surveys, in The Railroad Gazette, Jan. 9 and March 6, 1885; also communication from P. F. Brendlinger in issue of Feb. 6, 1885, and an exhaustive discussion of the subject by many engineers, November, 1884, to April, 1885, of same journal.

VI. *Conclusions.*

12. In regard to locating lines of railway, we think it will in general be granted, —

1°. That in very rough, or even in moderately hilly, country, a good contour map, if not a necessity, can greatly aid in making a good location.

2°. That a slight additional expense, to obtain a really accurate contour map, is always justifiable in rolling or hilly country.*

3°. That in the absence of a map, in a hilly country, many trial lines are apt to be abandoned before a location is effected, thus adding considerably to the expense of the survey, without finally getting the best line the ground afforded.

4°. That a good locating engineer can nearly always determine on the position of a line to give proper grades, within, say, 600 or 800 feet; that is, after the general route is determined, a belt not more than 1,200 or 1,600 feet in width could be located, which would certainly include the best line the ground afforded.

5°. That a belt of sufficient width to embrace the best line the ground affords, being, perhaps, never more than 1,500 feet wide, and generally not more than 600 feet wide, could be surveyed, and contours accurately determined, by a single party by the stadia system at about the same expense as usually attaches to an ordinary good preliminary survey employing one transit man, one leveller, one topographer, two chain men, one rod man, and two flag men. The leveller, the topographer, and the two chain men are dispensed with; and, inasmuch as it is not necessary that the stadia stations come in a straight line, the amount of cutting would not be very different.

6°. That the advantages of the stadia system, for the obtaining of reliable contours, with permanent field notes for

* This is a very mild statement, for generally a very considerable expense is justifiable in order to obtain such a map.

proving the same, can scarcely be compared with the awkward and unreliable methods of the hand level or clinometer on the one hand, or with the laborious and excessively expensive method of the engineer's level and chain on the other.

If the above premises be granted, then the following conclusion seems unavoidable : —

That most preliminary surveys for railroad lines might advantageously be made with the transit and stadia.

When the ground is level, or where the location is definite, as along the foot of a bluff or bank of a stream, the belt may be contracted in width to the needs of the final map. Certainly, for the obtaining of the data for the final charts of the company, no other method is so well adapted; and, since there never need be taken more data than will be of service, it would seem the system is all that could be desired for this purpose.

CHAPTER V.

OTHER APPLICATIONS OF THE STADIA SYSTEM.

1. *A Preliminary Survey for a Canal* would be quite similar to that for a railroad, except that the contours would be more rigidly followed, and perhaps more accurately determined. Otherwise, the problems here are very similar to those arising in the preliminary survey for a railroad.

2. *Surveys for Ditches and Pipe Lines* in mining regions or elsewhere should always be made in this way.

3. *Drainage Basins* for water supply, sewerage, or for ditch or tile drainage, are best surveyed by means of the stadia, for here the matter of contours is significant. Regions subject to overflow from reservoir or dam projects are also best determined by this means.

4. *Preliminary Surveys for Dams, Reservoirs, and Aqueducts* are especially adapted to the use of the stadia.

5. *Surveys for Bridge Sites,* bridge foundations, and all river and harbor work, where chaining is difficult or impracticable, should be done by the stadia.

6. *Surveys of City and Town Sites, Parks, and Cemeteries.* — Before the proper grades can be determined for a city or village, a general contour map is necessary, in order that the grade of the streets may fairly represent the average surface when this has been graded down to the plane of the streets. If the ground is very irregular, there is no other way of judiciously fixing the street grade. If the street lines alone are run out, and profiles drawn from which the grades are fixed, they will not usually represent the average grade.

Surveys of parks and cemeteries are usually made for the purpose of obtaining a topographical map of the same. The geographical positions are of more importance than the elevations, but it is evident that the method by stadia has great advantages over the transit (or compass) and chain. The cost of such a survey by the stadia should not exceed one-half the cost when the chain is used, and the results still be more accurate than could be shown on the map. Every city engineer's office should have an outfit of stadia boards to accompany one or more of the transits.

7. *Surveys of Mines.*—The stadia has been used to great advantage in the survey of mines, where chaining is inconvenient. Here the air is very steady, and there is nothing to disturb the instrument, so that very accurate readings may be obtained. The stadia rod must be short; and, instead of having it graduated in black and white, it is best to have a scale painted upon it, which is determined by observation, the same as the ordinary stadia, but which is not to be read from the instrument. The trouble is, to get sufficient light to read a painted board. To remedy this, have two polished strips, say a quarter of an inch wide, which can be held to the surface of the board. Let one be fastened with its top even with the zero of the stadia scale, and let the other be moved to suit the position of the other wire. When this is set, the reading of the top edge of the upper scale gives the distance. This is read off by the rodman, so that the reading can be taken as accurately as the disk can be set. If the plate or disk can be set to the nearest 0.005 of a foot, then the distance is obtained to the nearest six inches, provided the wires are so set that 1 foot on the rod corresponds to 100 feet in distance. A third man must illuminate the polished plates while the rodman adjusts the upper one. It might be found desirable to rig two lamps on a staff, one being fixed and the other movable. The position of the movable lamp would then give the distance.

8. *Geological Surveys.*—The system is especially fitted for this kind of work, for it is generally done in hilly or mountain-

ous regions, where chaining is both laborious and inaccurate, and where lines of levels up and down the sides of the hills, to obtain the contour lines, is not to be thought of. Here, too, the work can generally be checked upon triangulation stations, and on bench-marks determined by a level. In fact, geological surveys with accurate determination of contours is hardly possible without the use of the stadia rod.

9. It may well be asked, if the stadia is so well adapted to such varied uses, why it has not come into more general use with engineers and surveyors. There are several answers to this question. In the first place, the method is comparatively new in this country. The first publications on the subject in America, so far as we are aware, were in the "Journal of the Franklin Institute," January and February, 1865. These were probably read by but few engineers; and, besides, the articles were not sufficient to enable an engineer wholly unacquainted with the method to successfully carry it into execution. Since that date occasional magazine articles have appeared, all of which were, however, open to the same objection as offered above.

It has been very extensively used on government surveys, mainly through the influence of engineers who became acquainted with its use on the United States Lake Survey.

The State Geological Survey of Pennsylvania has covered some 3,000 square miles with a topographical survey by means of the transit and stadia. Mr. Rudolph Hering, C.E., has just completed a survey of some 400 square miles for the new water-supply drainage basin for the city of Philadelphia, and this was by the transit and stadia. Several State geological surveys have substituted the transit and stadia for the plane table and stadia, as being more expeditious, and more reliable in the matter of elevations.

10. Each new application of the system develops new methods of applying it. Thus, Mr. Hering, in place of making freehand sketches in his note-book, to assist in plotting the notes, mounts a drawing-board, with a sketching-sheet attached,

by the side of his instrument, and makes his sketch on this, plotting his readings as he takes them, getting the distance from the stadia reading, and sighting to the point across the board for his direction. He thus obtains many of the advantages of the plane table, without having to submit to the disadvantages.

On the Pennsylvania State Geological Survey, in the vicinity of working-mines, or where land is valuable, and private surveys and property lines numerous, the work is done with great care and accuracy. Much time and pains are taken on the field sketches, — so much, in fact, that it is often found injudicious to have more than one stadia man to a transit. In this case the party consists of but two men, one at the instrument and one with the stadia; and these often change places, in order to let the engineer go about with the rod, and make his sketches. A most remarkable accuracy is attained in this work as to both geographical position and elevation. The length of sight is limited to 400 feet, and needle bearings alone taken. When the instrument is moved, it is taken 400 feet *beyond the forward station*, so that only every second station is occupied by the instrument. This may be done where magnetic bearings are taken, as in ordinary compass work, but would not be feasible when running by backsights. There being no local attraction, the errors of the needle bearings nearly compensate on the numerous short courses, so that the resulting errors of position are small. If long sights were taken, as 1,200 to 1,500 feet, then considerable errors would arise from the uncertainties of the magnetic bearings.

CHAPTER VI.

GENERAL CONSIDERATIONS.

1. *Accuracy.* — Notwithstanding the amount of work that has been done in this country with the stadia, the only available data as to its accuracy is given by the report of the United States Lake Survey for 1875. The entire stadia work of that year was co-ordinated and compared with the corresponding distances as determined by the triangulation. One hundred and forty-one meandered lines, varying in length from a half-mile to four miles, the mean length being a mile and a half, were thus tested, with an average error of one in six hundred and fifty. The length of sight between stadia stations would probably average from 800 to 1,000 feet, with maximum distances of twice these amounts. The limit of allowable error, in closing on a triangulation station, was one in three hundred.

No especial pains were taken to make these lines more accurate than others, as it was not known at the time that this test was to be applied to them. The readings were taken to the nearest metre. The rods were graduated for a single distance, and no corrections were applied when the distance read was greater or less than this. The accuracy here attained was sufficient for the object of the survey, which was simply a general topographical representation of the shore line, for a distance of a mile from the lake shore, on a scale of 1 to 10,000, with 20-foot contours. If more care were exercised in the work, limiting the readings to 1,000 feet, and applying all corrections, it would be an easy matter to bring the error within 1 to 1,200. One advantage of the method lies in the fact

that the errors are all compensating; so that, by the law of accidental errors, only the square root of the number of errors remains uncompensated. In measurements by the chain, there are usually cumulative errors, such as an erroneous length of chain, sag, and measurements not on a level. Thus, any given short distance could be measured more accurately with a chain; but, when several miles are to be run, the accumulation of small errors in the use of the chain will usually exceed the residual uncompensated error from the use of the stadia. On hilly or obstructed ground, the stadia would probably give more accurate results on distances as short as half a mile. It will be seen, therefore, that the transit and stadia might well be used in many cases in land surveying, in place of the compass and chain.

2. *Cost.* — There is little available information on this subject. The topographical part of a survey is not usually kept entirely distinct from other work, so that its cost is not independently determined. The reports of the Mississippi River Commission furnish some evidence. The survey of that river has been completed from Cairo to Donaldsonville, a distance, by river, of 885 miles, in the last five years. This survey includes a belt of topography, on each side of the river, from a mile to a mile and a half in width, making, in all, something like 2,500 square miles of topography. The cost of the topography alone is not given; but the total cost of the field surveys is given from year to year, per square mile, including the hydrographic work on the stream itself. Thus, the cost per square mile of the whole belt covered, including the river and its soundings, is about sixty dollars. The cost of the topography alone would exceed this, and perhaps should be placed at about *seventy-five dollars per square mile.* In this work, the contours are five feet apart; and a large portion of the region is heavily timbered, through which lines have to be cut. The surface is also very irregular, being cut up by many bayous and old water-ways, which require the taking of a great many contour points. A permanent stone bench mark, with

some attachments to mark the spot, is also set about every square mile, and is included in the above cost. The cost of making the pencil field plots, to a scale of 1 to 10,000, is also included in the above, but not the final charts in ink, which are worked up in the office.

A sample of these maps is given in Plate I.

If the elevations and contours are not desired, the cost would be very much reduced, perhaps by more than fifty per cent.

In open country it would also be very much less.

3. In 1882 Mr. William Bell Dawson, C.E., made a survey, covering 180 square miles, by the transit and stadia, determining relative elevations in a general way, but not drawing contours. An account of this survey is given in the "Proceedings of the American Society of Civil Engineers for 1882," p. 397. The cost was only about fifteen dollars per square mile.

The survey of the banks of the Missouri River has been made with the stadia without taking contours, and the estimated cost is about twenty-five dollars per square mile.

CHAPTER VII.

COMPARISON OF METHODS OF TOPOGRAPHICAL SURVEYING.

1. THERE are, in general, three methods of making accurate topographical surveys; viz., —

By the transit and stadia.

By the plane table and stadia.

By the compass (or transit), level, and chain.

Of the various methods of obtaining approximate topographical representations, such as with the prismatic or pocket compass, the lock or hand level, the barometer, the clinometer, the pedometer, the odometer, etc., nothing will here be said: they find their proper use only in a *reconnaissance*. A record made with such means should not be called a survey.

2. The method of topographical surveying by means of the transit and stadia has already been described.

The method by means of the plane table and stadia is very fully described in the "Report of the United States Coast and Geodetic Survey for 1880," Appendix No. 13.* Some thirty pages of text, with many cuts, are there given, including the solution of many complex problems. The method is also taught in all our polytechnic schools, and is more or less fully treated in all text-books on surveying. It will therefore not be treated here, except to compare it with the method by the transit and stadia.

The method by means of the level, compass, and chain is

* Either this appendix or the entire report can be had on application to the superintendent of the survey, Washington, D.C.

so laborious, tedious, and expensive, that it is only warranted for very small areas, as a few acres of ground, and only then when other means are not available. It is safe to say, that a party of four could do the field work, with a given degree of accuracy, by means of the transit and stadia, in one day, over an area which would require a week to cover by means of the level, compass, and chain. This method, therefore, scarcely deserves mention as a means of general topographical surveying.

3. The method of topographical surveying, as herein described, can be properly compared, therefore, with but one other method; and that is by means of the plane table.

On the United States Coast and Geodetic Survey, the plane table has been exclusively used for making topographical surveys. The stadia* is used in connection with it. It is evident that the plane table was originally designed to be used in obtaining only geographical position, and that it is poorly adapted to obtaining accurate elevations. When only approximate elevations are desired, as where the elevations of the ground are shown by hachures instead of contours, or where the contours are fifty or a hundred feet apart, the plane table is still well adapted to the work. When, however, contours five feet apart are to be determined, and more especially when elevations must be carried from station to station over long lines with small residual errors, it is evident the plane table is not equal to the transit; for certainly the alidade cannot be shifted on the table without relevelling, as accurately as the transit may revolve on its vertical axis.

If the stadia be not used with the plane table, then points are only located by intersection; and this involves at least two readings on the same point from stations some distance apart. In obstructed country this is difficult to obtain.

Both of these methods of surveying have their respective advocates, who claim the one is superior to the other; and this

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* Called, on that service, a "telemeter."

is a question every engineer must decide for himself. Suffice it to say, both methods are complete in themselves, and competent for all ordinary topographical surveying. The plane table is especially adapted to the surveying of open and hilly country, where the view is unobstructed, and where many points are inaccessible. These points are located by intersections; but the same could be done with the transit, by reading azimuths to these points from two or more ⊡'s. If points are to be located by a single pointing and a stadia reading for distance, as must be done in wooded country, then it would seem the plane table has no advantage.

The plane table is much heavier, and is transported with more difficulty, than the transit. It takes longer to set it up and orient it than does the transit.

If the plane table is to be used mostly as the transit is used, for locating points by reading distance on a rod, and taking the vertical angle, then one might as well take a transit at once. The difference is as to the plot. In the one case the point is plotted at once, in the other it is plotted afterwards. So far as this is a question of economy, the advantage is in favor of the transit: for the time of the whole party is taken when the plotting is done in the field, while only the time of the observer and recorder is taken when plotted in camp; and even this does not add to the expense if it be done evenings and on rainy days. The advantage of having the sheet in the field, and so sketching in the features on the ground, is, perhaps, wholly offset by the disadvantages of having no record except the field sheet, and of occasionally getting this spoiled by storm and accident. When the transit is used, the map may also be taken into the field, and the plotting and sketching done on the ground if thought desirable. The objection is, that it takes too much valuable time. The observer's sketches in the note-book can be made to serve the same purpose.

But perhaps greater than any of the above objections to the use of the plane table, is the amount of experience necessary to enable one to use it effectively. When we add to this the

additional fact, that the plane table can be used for no other purpose except that of topographical surveying, causing few engineers or corporations to put themselves to the expense of obtaining one, we can then see why the familiar use of this instrument is, and will ever be, almost unknown except on such large works as national and State surveys. On the other hand, the same transit which is employed to make a stadia survey can be used for all other work where a transit is required. Besides, when the use of the transit is once mastered, it requires but little additional knowledge or experience to enable the engineer to do good topographical work with this instrument.

4. In conclusion, we may say, —

1°. The plane table is adapted to open country and long distances where no contours are to be determined, and where the stations occupied are comparatively few, as well as where a multiplicity of details are required.

2°. The advantage from plotting the work in the field is probably about balanced by its increased cost.

3°. All that can be done by the plane table may be as well done by the transit.

4°. When the view is more or less obstructed, so that the distances must be read on the stadia, and many stations occupied, and more especially in all cases where contour lines are to be determined with accuracy, the method of the transit and stadia is far superior to that of the plane table.

5°. The plane table can be used only for topographical work, and requires a large special practice; the transit may be used for various other purposes, and requires little special training in order to use it in this way.

CHAPTER VIII.

BASE LINE, TRIANGULATION, AND AZIMUTH.

I. *Base-Line Measurement.*

1. If the survey includes, or is based upon, a system of triangulation, it becomes necessary to measure one or more lines, called *base lines*, with great accuracy. For systems of primary triangulation, such as are used for determining the arc of a meridian for instance, the base line should be measured to an accuracy of one to one million; this being about the degree of accuracy attainable in the comparison of standards of length.

For secondary systems of triangulation, such as are used for ordinary geographical location, an accuracy of one to one hundred thousand is sufficient. For still smaller systems, called tertiary systems, used either for small regions or for filling in the intermediate region between primary or secondary stations, the base may not have an accuracy of more than one to ten thousand.

For measuring primary bases, very complex apparatus is used. The greatest source of error in such cases is the uncertainty in knowing the temperature of the unit of measure. No compensating apparatus has yet been devised which is really compensating *for all rates of change of temperature;* neither has any metallic thermometer, a part of which is the measuring-bar itself, been devised which always gives the true temperature of such bar.

It need scarcely be said, that any thermometer separate

from the measuring-bar itself, however reliable it may be in recording *its own* temperature, will not record the temperature of a separate though adjacent body, when this temperature is rapidly changing, as it does in the field, even though the apparatus be well protected from sun and wind.

The absolute length of the unit of measure is readily obtained, in terms of known standards, to one in a million, and the field work can be done, so far as the mechanical part is concerned, to one in ten million: but the uncertainties in knowing the field temperatures bring the error of the field work up to more than one in a million.

If an error of *one in a hundred thousand* may be allowed, as it may in secondary work, then the base may be measured with a steel tape, at a very moderate cost of time and labor.

For an error of *one in ten thousand*, the ordinary surveyor's chain may be used, being careful to know the true length of the chain, and marking the chain lengths on the tops of stakes driven in the ground; the chain to be carefully levelled and plumbed on irregular ground.

2 *Measuring a Base with a Steel Tape.*—It is not difficult to measure a base line of any desired length, with a steel tape, to a greater accuracy than *one in a hundred thousand*, provided proper precautions are taken. Here, too, the uncertainty in knowing the temperature of the tape will be the largest source of error. It will be here assumed that the absolute length of the tape, or, in other words, the temperature at which the tape is standard, is known. This standard temperature is usually stamped upon the tape.

The co-efficient of expansion may be taken at 0.000007 for 1° F.

The modulus of elasticity may be assumed to be 27,400,000,* in pounds, per square inch.

In measuring a base line with a steel tape, the mechanical

* These were the constants for a 300-foot steel tape which the author himself determined with great care.

errors may be reduced almost to zero, so that practically the only uncertainty will be in the matter of the temperature. This is done by suspending the tape by pendent hooks which are free to swing about their supports, thus wholly eliminating the friction of the tape on its supports. Then, for a given tension and temperature, we have always the same distance between end graduations. The error of marking the tape length, on a tack head for instance, is practically zero when compared to one tape length, which should not be less than three hundred feet. To make the error from temperature as small as possible, the work should be done on a *calm, cloudy day*, when earth and air are at the same temperature, so that there is no radiation. Several thermometers which have been carefully compared with some standard, and their errors determined, should be attached to the tape at intervals.

The method, in detail, is as follows:—

Having selected the line of the base, have stakes driven firmly, with one face on line, and with their tops approximately to grade. Decide upon a series of grades for the tape to occupy in the measurement, and see that these grades do not come nearer the ground than one foot at any point. Mark these grades by driving nails horizontally on grade in the sides of the stakes which have been set on line. From these nails suspend hooks of a uniform length of about two inches. It is preferable to have each tape length on one grade; but this is not essential, provided the change of grade on any tape length is not such as to lift the tape from any of its supports. With this exception, these grades may be any thing whatever, provided they are well determined. At every tape length provide a very solid marking-post, having its broad top on the grade of the tape when suspended in the hooks from the nails. On either side of this post, on line, at a distance of about two feet, set stakes of same height as the marking-post: these stakes are to serve the chain men as rests for their hands and tension scales. The stakes which support the tape should be a uniform distance apart, of from fifteen to fifty feet. If this distance is more

than about fifteen feet, the shortening of a tape length, due to the catenary effect, or sag, will be appreciable; and a correction must be introduced to compensate it.

The resting of the tape on swinging supports is an important one. The co-efficient of friction of iron on iron or wood is from three to six tenths: so that, if the tape weighs two pounds, the friction may be as much as one pound; that is, the tension on the tape may be one pound more at one end than at the other. The *mean* tension may then be one-half pound more or less than the *indicated* tension. For a 300-foot tape that weighs but two pounds, this would give rise to an error of two-thousandths of a foot (or six-tenths of a millimetre) for a pull on the tape of sixteen pounds.

If the best results are desired, the base should never be measured when the sun is shining, or when there is an appreciable wind. It might be well done at night, but this is not necessary if the work may be postponed for a favorable day. Having the stakes all prepared over the entire line, it will not take long to measure the base. A set of hooks may be prepared for all the stakes, and these hung once for all; or there may be one set of hooks sufficient to swing one tape length, and these carried along, and reset for each tape.

From the foregoing the method of doing the work is evident. The rear chain man holds his end of the tape to the proper mark, supporting his hands on the stake back of the marking or end stake. The tape must evidently be lengthened a couple of feet for this purpose, at each end. The object of these auxiliary stakes, either side of the marking-stake, is to avoid bringing any lateral strain upon the marking-stake, which is to remain fixed, in order to hold the work while carrying the tape forward. An assistant sees to lifting the tape into the hooks, and to the adjusting of these to a vertical position, after the head chain man has brought the requisite tension upon the tape by means of his spring balance. In using the balance, care must be exercised that the extension rod does not bear hard against its guides. When all is ready, the rear chain man

having the tape at the mark, and the head chain man having the proper tension, as read on his scales, the attendant marks, with a knife, the position of the forward graduation on the top of a copper tack driven in the head of the marking-stake. The thermometers, of which there should be at least three if the best results are desired, are then read and recorded, and the chain men move on. It is evident all this need not occupy more than about five minutes for a 300-foot tape and four men. Three men may do the work. A 500-foot tape is, perhaps, more desirable for this work. The tape should be small, or what is known as hoopskirt size, being of about 0.002 square inch in section. The suspension hooks should be from two to three inches long. If desired, a system of end fastenings may be arranged for attaching the tape directly to the pulling-stakes by adjustable hooks or clamps, and a slow-motion arrangement, so as to dispense with the more or less unsteady muscular effort in stretching the tape. A pull of from eight to sixteen pounds will be found desirable.

The base should be measured two or three times, and the mean taken.

3. *Correction for Sag.* — Where the sag is small, as it always is in this work, the curve, although a catenary, may be considered a parabola without an appreciable error.

Let $w =$ weight of tape per running foot,
$d =$ distance between supports,
$P =$ horizontal pull on tape,
$v =$ sag midway between supports,
$L =$ length of the base;

w and P being taken in the same unit of weight, and d, v, and L in the same unit of length.

If we pass a section through the tape midway between supports, and equate the moments of the external forces on one side of this section, we obtain, taking centre of moments at the support,

$$Pv = \frac{wd}{2} \cdot \frac{d}{4} = \frac{wd^2}{8},$$

or

$$v = \frac{wd^2}{8P}. \tag{1}$$

If the length of a parabolic curve be given by an infinite series, and if all terms after the second be omitted, which they may when $\frac{v}{d}$ is small, then we may write

$$\text{Length of curve} = d\left(1 + \frac{8}{3}\frac{v^2}{d^2}\right). \tag{2}$$

If we now substitute for v its value as given in equation (1), we have

$$\text{Length of curve} = d\left\{1 + \frac{1}{24}\left(\frac{wd}{P}\right)^2\right\}.$$

If we call the excess in length of curve over the linear distance between supports the *correction for sag*, we have

$$\text{Correction} = c = \frac{d}{24}\left(\frac{wd}{P}\right)^2. \tag{3}$$

But this is the correction for a single sag. If L is any length of base, there will be $\frac{L}{d}$ such sags in the whole measurement; or, *the correction for the entire line is*

$$C = \frac{d}{24}\left(\frac{wd}{P}\right)^2 \frac{L}{d} = \frac{L}{24}\left(\frac{wd}{P}\right)^2. \tag{4}$$

Example.—Taking the constants of the 300-foot tape referred to above, we have

$w = \frac{18846}{300}$ grains $= 0.0066$ pound,
$d = 10$ feet,
$P = 10$ pounds,
$L = 1{,}000$ feet:

Then $C = 0.00181$ foot $= 0.55$ millimetre.

The relative error here, from this cause, if the correction is not applied, is 0.0000018, or 1 in 550,000.

From equation (4) we see that this correction varies as the square of the distance between supports, inversely as the square of the pull on the tape, and directly as the length of the base. If, therefore, our base had been 5,280 feet long, the distance between supports 50 feet, and the stretch on the tape 15 pounds, the correction would have been

$$5.28\left(\frac{5}{1.5}\right)^2(0.00181) = 0.106 \text{ foot.}$$

If the supports had been 25 feet apart, this correction would have been one-fourth as great, or 0.026 foot. In this case the relative error, if this correction were neglected, would be 0.00005, or 1 in 20,000.

It is evident that this correction is applied negatively to the measured length; or, in other words, the measured length is too great by the amount of this correction.

4. *Summary of Corrections in Measurements with the Steel Tape.*—In the following formulæ the corrections are to be applied to the measured length, with the signs as given.

1°. *Correction for Temperature.*

$$C_t = +0.000007(T - T_0)L,$$

where T is the mean temperature (Fahrenheit) of the tape for the given measurement, T_0 is the temperature at which the tape is standard, and L is the measured length.

2°. *Correction for Grade.*

$$C_g = (l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 + \text{etc.}) - L,$$

where L is measured length, and l_1, l_2, l_3, etc., are the lengths of the several portions whose angles with the horizontal are respectively θ_1, θ_2, θ_3, etc.

3°. *Correction for Sag.*

$$C_s = -\frac{L}{24}\left(\frac{wd}{P}\right)^2,$$

as given in equation (4) above, where L is measured length, w is the weight, in pounds, of one foot of the tape, d is the distance between supports in feet, and P is the pull of the tape in pounds.

4°. *Correction for Pull.*

$$C_p = +\frac{(P - P_0)L}{SE},$$

where P is pull on tape in pounds, P_0 is the pull when tested for standard, and may have been zero, L is the length of the base in *inches*, S is the area of the cross-section of the tape in square inches, and E is the modulus of elasticity of the tape in pounds to the square inch. The functions L and S must here be taken in inches, because E is always given in terms of that unit. The value of E as given above is 27,400,000. S could probably be determined with sufficient accuracy by an ordinary wire gauge which reads by vernier to thousandths of an inch. If this is not accessible, it could be determined by the aid of a three-legged transit or level. Thus, with the instrument dismounted from its tripod, set it upon a flat metallic surface, and level it up. With the telescope in the plane of one of the legs, read upon an object at a known distance from the line passing through the other two supports. Then carefully raise the leg under the telescope, without turning the levelling-screws in the least, and insert the tape under the foot of this screw. Then read again on the distant object. Knowing the distance from the foot of this screw to the line passing through the other two, and also the distance to the distant object, and the movement of line of sight on that object, the thickness of the tape can be computed very accurately.

A better method of determining the correction C_p is by stretching the tape by different weights, and observing the different extensions. In this case the tape should rest on a level floor or support, as the top of a rail on a railway track.

II. *The Triangulation.*

5. What is here given is intended to apply only to a secondary or a tertiary triangulation scheme, or such as may be made the basis of a topographical survey. The lengths of the sides of the triangles of such a system may be from one to ten miles. The stations should be selected so as to bring the lines of sight as high above the intervening ground or water as possible, both to save clearing out and to avoid the unsteady atmosphere. If stations are built on which to set the instrument, they would probably not be more than ten feet high, but should be very firm. The platform for the observer must be entirely separate from the instrument tripod. If no framed stations are constructed, either a post may be planted, on which to set the instrument, or perhaps a tree, properly located, may be cut off at the right height. The instrument preferably having three levelling-screws is set upon a trivet which is fastened to the top of the post by means of long sharp projections below, which are driven into the wood. Good work cannot be done with the instrument mounted on an ordinary tripod. The post may be whitewashed, and a hole bored vertically in the centre of the top, into which a whitened stick may be driven for the target when readings are taken to this station. If the stations are too far apart for this to serve, a very good target may be made of wire and canvas. One great fault of many targets is, that they offer a phase when the sun is shining; thus causing the pointing to be taken to one side of the centre. All targets having three dimensions are subject to this fault.

The sources of error in this kind of work are so numerous and various that it is hard to classify them. Most of them will, however, be covered in the course of this chapter.

6. *The Instrument.*—A transit (or theodolite) with a six-inch circle, if well graduated, may serve the purpose. It should read by vernier to ten or twenty seconds. Since no limb can be exactly centred and graduated, the observations must be so arranged as to eliminate eccentricity and systematic

errors of graduation. The telescope should be inverting, for clearer vision. The objective should be from one and one-fourth to one and one-half inch in diameter. The magnifying power of the eye-piece should be from thirty to fifty diameters. There should be three levelling-screws, and the plate bubbles should be sensitive and well set. The circle passing through the three supports should be about six inches in diameter. The mounting should be such as to permit the instrument being readily dismounted from its tripod, and set upon a trivet, without interfering.

If the same transit is to serve both for the triangulation and for the topographical work, the limb may be graduated to thirty seconds: there may be four levelling-screws if desired, but in this case the instrument would probably be always used upon its tripod.

7. *The Target.* — A most efficient target for lines from one to ten miles in length is made as follows: Bend three galvanized iron wires, about three-sixteenths inch in diameter, into circles of, say, six inches diameter, and fasten them by twisting or soldering. Attach to these three wire circles four wires about three or four feet long, making a frame, with the long wires as the four uprights. All joints had best be securely soldered. The rigidity of the frame will then depend only on the size and stiffness of the wires. These should increase, of course, with the size of the target desired. The target is divided into two or more zones by stretching black and white canvas alternately between the opposite uprights, making diametral sections. If are there ~~is~~ more than two zones, those marked by the same color should have the canvas crossing in different ways; so that, if one plane is nearly parallel to any line of sight, the other plane of this color will be at right angles to it. The black and white are both used, in order that the target may be visible against any background.

The great advantage of this target is, that it has no phase. If the pointing be always made to the centre of the visible portion, it will be made to the centre of the target. It is

mounted over the station by centring it at bottom, and supporting it in a vertical position by means of wire guys leading from its top to pegs in the ground. This target has been used on the Mississippi River Survey with excellent results.*

8. *Programme of Observations.* — The observations should be so made as to eliminate as many of the instrumental errors as possible.

Thus, the error of eccentricity is eliminated by reading both verniers.

The error in the adjustment of the transit axis to a horizontal position is eliminated by transiting the telescope.

The systematic errors of graduation of the horizontal limb are eliminated by reading each angle on different parts of the limb.

The error from a change in azimuth of the limb of the instrument itself, either from a slight slipping on its axis or from a twist of post or station, is eliminated by reading, first around to the right, and then around to the left.

The programme given below is arranged with these ends in view. The instrument is supposed to be graduated continuously to 360°. Suppose there are five stations to be read, beginning with A, and, by counting around to the right, closing on E. When reading towards the right, read on A, B, C, D, E, and then turn the telescope on beyond E to the right a short distance, and bring it back to E by moving towards the left, and read E, D, C, B, A. If these stations are not all at the same elevation with the instrument station, and if the transit axis is not exactly at right angles with the vertical axis of the instrument, then some of the angles have been read too large, and others too small, from this cause. If, now, the telescope be transited, and the same readings repeated, the angles that were first read too large will now be read too small by the same

* It was designed by John Eisenmann, C.E., in 1881, then United States Assistant Engineer, now Professor of Civil Engineering in the Case School of Applied Science, Cleveland, O.

amount, and *vice versa*. This should therefore be done, making four readings for each angle, and completing one *set* of readings. As many such sets may be taken as are sufficient for the degree of accuracy desired. If four such sets are to be taken, then *the limb should be shifted one-fourth of 180°*, or *45°*, between each set, and in the same direction. This will result in reading each angle on eight symmetrical parts of the circle, thus eliminating any systematic errors in the graduation of the limb. If n sets of readings are to be taken, then the limb is shifted $\frac{180}{n}$ degrees each time.

PROGRAMME.

1st Set.	2d Set.
Telescope normal.	*Telescope inverted.*
Read to right.	Read to right.
Read to left.	Read to left.
Telescope inverted.	*Telescope normal.*
Read to right.	Read to right.
Read to left.	Read to left.
Shift the Limb.	*Shift the Limb.*

Evidently each set is complete in itself; and as many complete sets may be taken as desired, but no partial sets should be used. If the work is interrupted in the midst of one set of readings, the partial set of readings should be rejected, and, when the work is resumed, another set begun. In reducing the work, if one reading of any angle is so erroneous as to have to be rejected, this should vitiate that entire set of readings of that angle.

9. *Reduction of the Notes.*—The mean of the two vernier readings of each angle is first taken out, and then the angles *A–B*, *B–C*, etc., in the note-books. The mean value of each angle is then found for each *set* of readings, and these tabulated for comparison. It is only these mean values for the several sets that should be expected to agree very closely. These should differ, aside from the accidental errors of observation,

only by the systematic errors of graduation on the limb, which would, of course, be very small. If these mean values are found to be very accordant for one of the angles of any triangle, but very discordant for another angle of the same triangle, then we may assume that the angle having the accordant results was better measured than the one with discordant results. If each of the angles of any triangle had been exactly measured, and if the instruments and targets had been accurately centred, then the sum of the angles should equal 180°.* In this case the triangle is said to close. In practice, a triangle never does close exactly. The common rule is to distribute the error of closure equally between the three angles, provided they have been equally well observed. These angles are usually considered to be equally well observed when they have all been measured the same number of times. This is weighting the observations by the external or *a priori* evidence. When the mean values of these angles are taken out, however, for the various sets of observations, we find that they were not all observed to the same degree of accuracy. If we should conclude from this that the error of closure should not be distributed uniformly, but the greater correction applied to the angle which was apparently the poorest observed, then this would be weighting the observations by the internal or *posteriori* evidence. If a series of triangles are adjusted only by distributing the error of closure, the latter method should be chosen.

10. *The Adjustment of a Triangle.*—In the triangle ABC, let A', A'', A''', etc., be the several mean values of the angle A for the different *sets* of observations, and similarly for B and C.

Let A_0 be the mean value of A', A'', A''', etc. Let v_a', v_a'', v_a''', etc., be the residuals corresponding to $(A_0 - A')$, $(A_0 - A'')$, $(A_0 - A''')$, etc.

Let Σv_a^2 be the sum of the squares of the A residuals.

* For such work as is here described, the question of spherical excess need not be considered; this being only 1 second of arc for 75 square miles.

Let c_a be the final correction to be applied to A_0 to give the most probable value of the angle A.

Then, when a similar notation is adopted for the angles B and C, we would have, by the principles of least squares,

$$c_a : c_b : c_c :: \Sigma v_a^2 : \Sigma v_b^2 : \Sigma v_c^2,$$

or

$$c_a : c_b : c_c : c_a + c_b + c_c :: \Sigma v_a^2 : \Sigma v_b^2 : \Sigma v_c^2 : \Sigma(v_a^2 + v_b^2 + v_c^2).$$

But $c_a + c_b + c_c =$ the error of closure of the triangle, which we may call E; and $\Sigma(v_a^2 + v_b^2 + v_c^2)$ is the sum of the squares of all the residuals for the three angles, which we may call V.

We may therefore write the three proportions

$$c_a : E :: \Sigma v_a^2 : V,$$

$$c_b : E :: \Sigma v_b^2 : V,$$

$$c_c : E :: \Sigma v_c^2 : V,$$

from which the three corrections c_a, c_b, and c_c are found.

11. *The Adjustment of a Quadrilateral.* — If four stations, A, B, C, D, be so situated that each may be seen from all the others, the system is called a quadrilateral, both diagonals having been observed. Here there are two sets of triangles, according to which one of the diagonals is used to divide the quadrilateral. It is evident, if one set of triangles be adjusted by any method so as to sum up 180°, that, when the other diagonal is taken, these two triangles would not, in general, close. If both diagonals are observed, then each of the four triangles is made up of four independently observed angles; and, if these be given equal weight, a very simple method may be found for adjusting the quadrilateral so that all four triangles will close, and this, too, with the most probable corrections for each angle.

It is shown in the method of least squares, that the most probable value of an observed quantity is such as makes the sum of the squares of the residuals a minimum. But this is only another way of saying, that the most probable value is the properly weighted arithmetic mean; since the arithmetic mean always does give the sum of the squares of the residuals a minimum. If the four angles of any triangle of the quadrilateral be given equal weight, then each of them should be corrected by one-fourth of the error of closure; for we have two determinations of the value of each angle, — one from measuring it directly, and one from measuring the other three, and subtracting their sum from 180°. This latter is no less a measurement of the angle because it is indirect. The direct measurement should, however, be entitled to three times the weight of the indirect method; since there were three chances for error by the indirect method to one by the direct method. Therefore, if V' be the observed value of the angle, and $V' + v$ the value by indirect measurement, the properly weighted arithmetic mean would be

$$V = \frac{3V' + V' + v}{4} = V' + \frac{1}{4}v.$$

The most probable correction to this angle, on the hypothesis that all the angles should be entitled to equal weight, is therefore one-fourth of the error of closure. On the same hypothesis, when there are but three observed angles in the triangle, the most probable correction to each angle is evidently one-third the error of closure.

To adjust the quadrilateral, we may first adjust the triangles *ABC* and *ADC* by distributing the errors of closure equally among the four angles of the respective triangles.

Then the triangles *ABD* and *CBD* must be adjusted so as to leave the sum of the angles in *ABC* and *ADC* unchanged. But, after the first set of triangles is adjusted, the sum of the eight angles of the quadrilateral will equal 360°, for they are composed of two sets, each equalling 180°.

Therefore, if the error of closure of ABD *is positive, the error of closure of* CBD *is negative by the same amount.*

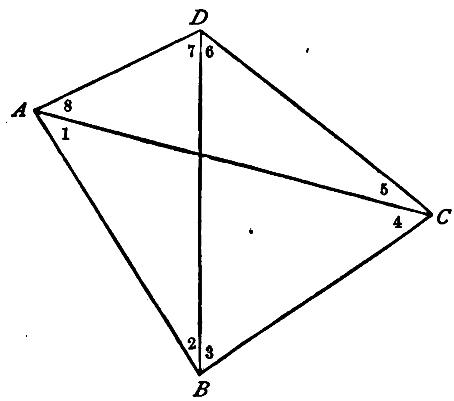

FIG. 7.

We have, therefore, to adjust these two triangles, ABD and CBD, by adding to the four angles of the one, and subtracting from the four angles of the other, one-fourth of the error of closure of these two triangles; or, in the second adjustment, the angles 1, 2, 7, and 8 will be treated alike, and 3, 4, 5, and 6 alike. In the first adjustment, the angles 1, 2, 3, and 4 were served alike, and 5, 6, 7, and 8 alike. Therefore the total correction made to the angle A_1 equals that made to B_2, that made to B_3 equals that made to C_4, etc.; so that, if these final corrections be denoted by a_1, b_2, b_3, c_4, etc., we will have

$$a_1 = b_2, \qquad b_3 = c_4, \qquad c_5 = d_6, \qquad \text{and} \qquad d_7 = a_8.$$

Also, we see that if in the second adjustment the angles 1, 2, 7, and 8 are corrected by the same amount as the angles 3, 4, 5, and 6, but with opposite signs, then the change made in 1 and 2 compensates the change made in 3 and 4, so that the triangle ABC will still sum up 180°. The same would be true of the triangle ADC.

We might, therefore, have the

Rule: Adjust two opposite triangles of the quadrilateral by distributing the error of closure of each equally between the four angles of that triangle. Then, with the new values of the angles in the other two triangles, distribute the error of closure as before, and this will bring all four triangles to sum 180°. This may be done more readily, however, at one operation, as given below.

If the original errors of closure of the triangles *ABC*, *ABD*, and *ADC* be v_1, v_2, and v_3 respectively, then by the first adjustment of the triangles *ABC* and *ADC* we made

$$a_1' = b_2' = b_3' = c_4' = -\tfrac{1}{4}v_1,$$

and

$$c_5' = d_6' = d_7' = a_8' = -\tfrac{1}{4}v_3.$$

The original error of closure of *ABD* was v_2; but, after the first adjustment, this error has come to be $v_2 - \frac{1}{2}v_1 - \frac{1}{2}v_3$: and it is minus one-fourth of this amount (the signs changed, since the *correction* is always of the opposite sign from the error) which is applied to the angles 1, 2, 7, and 8, and plus one-fourth this amount applied to the angles 3, 4, 5, and 6.

The second corrections are, therefore,

$$a_1'' = b_2'' = d_7'' = a_8'' = -\tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3),$$

and

$$b_3'' = c_4'' = c_5'' = d_6'' = +\tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3).$$

Combining these two sets of corrections for the total correction, for the several angles we have

$$a_1 = (a_1' + a_1'') = b_2 = (b_2' + b_2'') = -\tfrac{1}{4}v_1 - \tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3),$$

$$b_3 = (b_3' + b_3'') = c_4 = (c_4' + c_4'') = -\tfrac{1}{4}v_1 + \tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3),$$

$$c_5 = (c_5' + c_5'') = d_6 = (d_6' + d_6'') = -\tfrac{1}{4}v_3 + \tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3),$$

$$d_7 = (d_7' + d_7'') = a_8 = (a_8' + a_8'') = -\tfrac{1}{4}v_3 - \tfrac{1}{4}(v_2 - \tfrac{1}{2}v_1 - \tfrac{1}{2}v_3).$$

These results may be put in the form,

$$a_1 = b_2 = \frac{-v_1 - 2v_2 + v_3}{8},$$

$$b_3 = c_4 = \frac{-3v_1 + 2v_2 - v_3}{8},$$

$$c_5 = d_6 = \frac{-v_1 + 2v_2 - 3v_3}{8},$$

$$d_7 = a_8 = \frac{+v_1 - 2v_2 - v_3}{8}.$$

From these formulæ the corrections for the several angles may be at once written out and applied, when all four of the triangles should sum 180°.

If the method of two successive adjustments is employed, as given in the rule above, then it is evident that the first set of triangles could be adjusted by weighting the corrections as the sum of the squares of the residuals, as given for a triangle with three measured angles. In the adjustment of the two remaining triangles, however, the angles should be given equal weight, in order not to effect the closure of the two triangles already adjusted.

12. *Computing the Triangles.* — After the angles of the system are adjusted, the sides of the triangles are computed by the ordinary sine ratio for plane triangles.

If the system consists of simple triangles, then one side is known, and the other two sides computed from it; there being no check on the work, except what the computer carries along with him, or what may be obtained by a duplicate computation.

If the system consists of a series of quadrilaterals, then the base line which is common to two adjacent quadrilaterals is computed through two sets of triangles from the previous known side. Thus, if the quadrilateral of Fig. 7 is one of a series, two of whose common bases are *AB* and *DC*, then *AB* has been computed in duplicate from the previous quadrilateral, and the mean of the two values taken. Now, in the triangle

ABD compute *AD*, and then in the triangle *ADC* compute *DC*; in the triangle *ABC* compute *BC*, and then in the triangle *BCD* compute *DC* again. We now have two independent values of *DC* as coming from *AB*. Take the mean of these two values as the length of *DC*, and proceed with the next quadrilateral.

It is to be observed, that, although we applied the *most probable* corrections of the angles to the quadrilateral, we of course did not apply the *absolutely true* corrections, and therefore the *adjusted* angles are still not the *true* angles. Therefore the two values of *DC* as computed from *AB* through the two sets of triangles will not exactly agree, but their difference should be very small. Large errors in the computation can be checked in this way, but small ones cannot be.

13. *Computing the Geodetic Positions.* — After the angles of the system are adjusted, and the sides of the triangles computed, we have the plane angles and linear distances from point to point in the system. It now remains to compute the latitudes and longitudes of the several stations, and the azimuths of the lines. This may be done in two ways. We may first co-ordinate the work, considering the stations all in one plane; that is, compute the latitudes and departures of the several stations referred to one station as the origin, and then compute the latitudes, longitudes, and azimuths from these co-ordinates, thereby referring each station, and the direction of the meridian through it, directly to the known point. Or, we may compute these three functions directly from the angles and distances of the triangulation system, thus referring each station to the preceding one. Both these systems are used, the former being best adapted to the determination of the geodetic positions for mapping-purposes, and the latter for determining the length and azimuth of the several lines of the system for the use of a topographical survey.

The following formulæ, though not exact, are quite sufficient when the sides of the triangles do not exceed ten or fifteen miles in length.

Notation.

Let L = the latitude of the known point,
L' = the latitude of the unknown point,
M = the longitude of the known point,
M' = the longitude of the unknown point,
Z* = the azimuth of the unknown point from the known, (forward azimuth),
Z'* = the azimuth of the known point from the unknown, (back azimuth),
k = the length of line joining the two points,
u'' = this length in seconds of arc of a great circle,
y and x = the latitude and departure respectively of the unknown point referred to the known,
e = eccentricity of earth's meridian section,
N = length of the normal, or the radius of curvature of a section perpendicular to the meridian at the middle latitude, in yards,
R = the radius of curvature of the meridian, in yards.

Then we have

$$\left.\begin{aligned} L' &= L - (1 + e^2\cos^2 L)\frac{k}{N\sin 1''}\cos Z, \\ M' &= M + \frac{k}{N\sin 1''}\frac{\sin Z}{\cos L'}, \\ Z' &= 180 + Z - \frac{k}{N\sin 1''}\frac{\sin Z}{\cos L'}. \end{aligned}\right\} \qquad (1)$$

If we let $A = \dfrac{1}{N\sin 1''}$, these equations may be written

$$\left.\begin{aligned} &\text{Diff. of latitude} = \Delta L = L - L' \\ &\qquad = Ak(1 + e^2\cos^2 L)\left[\cos Z + (\tan L \tfrac{1}{2}\sin 1'')Ak\sin^2 Z\right]. \\ &\text{Diff. of longitude} = \Delta M = M' - M = Ak\frac{\sin Z}{\cos L'}. \\ &\text{Diff. of azimuth} = \Delta Z = Z' - Z = 180° - \Delta M \sin \tfrac{1}{2}(L + L'). \end{aligned}\right\} \qquad (2)$$

* Z is measured continuously from the south point in the direction S.W.N.E. Due attention must be paid to the signs of the Z functions.

The value of e^2 is 0.00667435, and

$$\log e^2 \quad + 10 = 7.8244105.$$

$$\log \sin 1'' + 10 = 4.6855749.$$

Between the latitudes of 30° and 50° we have, for the value of $\frac{1}{N \sin 1''}$, the following: —

Latitude.	$\log A + 10$ $= \log \frac{1}{N \sin 1''} + 10.$	Diff. for 1°.
30°	8.4705474	−220
35°	8.4704327	−239
40°	8.4703104	−250
45°	8.4701842	−254
50°	8.4700579	−251

In the last of equations (2) the term $\Delta M \sin \frac{1}{2}(L + L')$ represents the *convergence of the meridians* between the two stations. This may be stated verbally thus: *The convergence of the meridians between two stations is their difference of longitude into the sine of their mean latitude.*

To compute the L's, M's, and Z's when the rectangular co-ordinates are known, we have

$$x^* = k \sin Z, \qquad y^* = k \cos Z,$$

$$\Delta L = L' - L = \frac{y}{R \sin 1''} - \frac{1}{2} \sin 1'' \left(\frac{x}{N \sin 1''}\right)^2 \left[\tan\left(L + \frac{y}{R \sin 1''}\right)\right];$$

or, if $\frac{1}{R \sin 1''} = B$, we may write

$$\left.\begin{aligned} \Delta L &= By - (Ax)^2 \tfrac{1}{2} \sin 1'' \tan(L \pm By), \\ \Delta M &= M' - M = \left(\frac{x}{N \sin 1''}\right)\frac{1}{\cos L'} = \frac{Ax}{\cos L'}, \\ \Delta Z &= Z' - Z = 180° - \frac{x \tan L'}{N \sin 1''} = 180° - Ax \tan L'. \end{aligned}\right\} \quad (3)$$

* Due attention paid to signs of sin Z and cos Z.

The following are logarithmic values of $B = \dfrac{1}{R \sin 1''}$ between 30° and 50° of latitude.

Latitude.	$\log B + 10$ $= \log \dfrac{1}{R \sin 1''} + 10.$	Diff. for 1°.
30°	8.4727305	−660
35°	8.4723864	−716
40°	8.4720194	−750
45°	8.4716408	−762
50°	8.4712619	−750

When the sides of a system of triangulation have been computed, and the azimuths of the lines are desired from the several stations, the successive differences of latitude and longitude are first computed, and from these the azimuths of the lines, using equations (2). The last of these equations gives the difference between the forward and back azimuth of the line joining the two stations. This difference being applied, with the proper sign, gives the azimuth of the first station as seen from the second. But when the azimuth of one line from a station is known, the azimuths of all other lines from that station may be found from the adjusted plane angles at that station. The back azimuths of these new lines being then determined in a similar manner, the azimuths of the various lines radiating from this second order of stations are found, and so on.

III. *To find Azimuth from Circumpolar Stars.*

14. For the purposes of a topographical survey, the azimuth of any line need not be known nearer than about one-half minute. For this degree of accuracy a single observation on a circumpolar star at elongation is sufficient, provided the instrument is known to be in good adjustment.

The azimuth of a star at elongation is found from the formula,

$$\text{Sine of azimuth} = \frac{\text{sine of polar distance}}{\text{cosine of latitude}}.$$

This formula is so simple that it is hardly necessary to give a table of values of the azimuth of Polaris, for example, for various latitudes. The polar distances of two close circumpolar stars, whose elongations occur about five hours and thirty minutes apart, will be given, from 1885 to 1930, from which their azimuths at elongation may be readily computed for any latitude.

The two stars selected are Polaris and 51 Cephei. Their relative position is shown in Fig. 8.

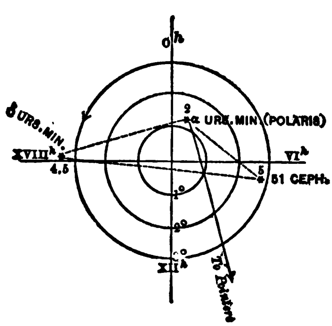

FIG. 8.

The position of 51 Cephei, which is of the fifth magnitude, may be described with reference to the line joining "the pointers," in the constellation of the Great Bear, with Polaris. Thus, 51 Cephei is to the *right* of this line, when looking *towards* the pole star along the line, at a distance of about *three times* the sun's disk from the line, and of about *five times* the sun's disk from Polaris, in the direction of the pointers.

In case 51 Cephei is not visible to the naked eye, as it may not be on moonlight nights, or with a slightly hazy atmosphere,

it may be found, when near elongation, by the telescope, as follows: —

Having carefully levelled the instrument, turn upon Polaris. When 51 Cephei is near its eastern elongation Polaris is near its upper culmination, and when near its western elongation Polaris is near its lower culmination. To find 51 Cephei at eastern elongation, therefore, after taking a pointing on Polaris, lower the telescope (diminish the vertical angle) by about one degree (if the time is about twenty minutes before elongation), and then turn off towards the east about two and a half degrees. This will bring the cross wires approximately upon the star.

To find it at western elongation, simply reverse these angles; that is, increase the vertical angle one degree, and turn off to the west two and one-half degrees.

The following table gives the times of the elongations of these two stars for 1885 for latitude 40°, which may be used for observing azimuth: —

TIME OF ELONGATION, 1885. LATITUDE, 40°.

Date.	Polaris.		51 Cephei.	
	Elongation.	Time.	Elongation.	Time.
Jan. 1 . .	Western	$12^h\,24^m.6$ A.M.	Western	$5^h\,48^m.3$ A.M.
Feb. 1 . .	"	$10^h\,22^m.2$ P.M.	"	$3^h\,46^m.4$ "
March 1 . .	"	$8^h\,31^m.8$ "	"	$1^h\,56^m.1$ "
April 1 . .	"	* $6^h\,29^m.7$ "	"	$11^h\,54^m.0$ P.M.
May 1 . .	Eastern	* $4^h\,36^m.6$ A.M.	"	$9^h\,55^m.9$ "
June 1 . .	"	$2^h\,37^m.0$ "	"	* $7^h\,53^m.9$ "
July 1 . .	"	$12^h\,39^m.0$ "	Eastern	* $6^h\,12^m.6$ A.M.
Aug. 1 . .	"	$10^h\,38^m.1$ P.M.	"	$4^h\,10^m.8$ "
Sept. 1 . .	"	$8^h\,36^m.6$ "	"	$2^h\,09^m.1$ "
Oct. 1 . .	"	* $6^h\,38^m.9$ "	"	$12^h\,11^m.4$ "
Nov. 1 . .	Western	$4^h\,26^m.4$ A.M.	"	$10^h\,09^m.8$ P.M.
Dec. 1 . .	"	$2^h\,28^m.2$ "	"	$8^h\,12^m.0$ "

From the above table, it is evident that an elongation of one of these stars may always be obtained.

* Probably not visible to the naked eye.

For other days than those given in the table, either interpolate, or find by allowing $3^m.94$ for one day, remembering that each succeeding day the elongation occurs *earlier* by this amount.

For other years than 1885, take from the table the time corresponding to the given month and day, and add $0^m.35$ for each year after 1885; also,

Add 1^m if the year is the second after leap year.
Add 2^m if the year is the third after leap year.
Add 3^m if the year is leap year before March 1.
Subtract 1^m if the year is leap year after March 1.

For the first year after leap year, there is no correction except the periodic one of $0^m.35$ per annum.

For other latitudes than 40°, add $0^m.14$ for each degree south of 40° north latitude, or subtract $0^m.18$ for each degree north of 40° north latitude.

The following table gives the pole distances of Polaris and 51 Cephei for Jan. 1 of each third year from 1885 to 1930: —

POLE DISTANCE (90° — DECLINATION).

Star.	1885.	1888.	1891.	1894.	1897.	1900.	1903.	1906.
Polaris . . .	1° 18′ 16″	1° 17′ 19″	1° 16′ 23″	1° 15′ 26″	1° 14′ 30″	1° 13′ 33″	1° 12′ 37″	1° 11′ 41″
51 Cephei . .	2° 46′ 35″	2° 46′ 47″	2° 47′ 00″	2° 47′ 13″	2° 47′ 26″	2° 47′ 40″	2° 47′ 54″	2° 48′ 09″

Star.	1909.	1912.	1915.	1918.	1921.	1924.	1927.	1930.
Polaris . . .	1° 10′ 45″	1° 9′ 49″	1° 8′ 53″	1° 7′ 58″	1° 7′ 2″	1° 6′ 7″	1° 5′ 12″	1° 4′ 16″
51 Cephei . .	2° 48′ 24″	2° 48′ 39″	2° 48′ 55″	2° 49′ 12″	2° 49′ 28″	2° 49′ 45″	2° 50′ 1″	2° 50′ 18″

To find the pole distance for any intermediate time, make a linear interpolation between the two adjacent tabular values.

The azimuth of the star at elongation is then found from the equation on p. 82.

15. In case the latitude of the place is not approximately known, it may be found readily to the nearest minute by an observation on one of the two stars here used, at either upper or lower culmination. The time of these culminations may be found from the times of elongations by knowing that an upper culmination occurs $5^h\ 59^m$ after an eastern elongation or before a western elongation, and that a lower culmination occurs $5^h\ 59^m$ after a western elongation and before an eastern elongation. If an observation be taken on a star at upper culmination, then the latitude is found by subtracting the star's pole distance from its altitude. If observed at lower culmination, add its pole distance to its altitude for the latitude of the place.

Since the two stars here used are $5^h\ 30^m$ apart in time, one is always near culmination when the other is at elongation, so that observations for azimuth on one may always be conveniently coupled with an observation for latitude on the other. Thus, when Polaris is observed at elongation, 51 Cephei will be observed at culmination about thirty minutes earlier; while if 51 Cephei is observed at elongation, Polaris may be observed at culmination thirty minutes later. This relation is clearly shown in Fig. 8, the motion of the stars being opposite to that of the hands of a watch, as indicated by an arrow point on the outer or three-degree circle.

If the plate levels and the vertical circle are in good adjustment, the latitude can be obtained in this manner to the nearest minute, or even closer with a good instrument.

If observations be made on two stars at elongation, then we have two such equations as given on p. 82, in each of which the cosine of the latitude enters. We may now eliminate this term, and get one equation which will give the azimuth independent of the latitude. It is best, however, to observe for the latitude, since it is so readily done (except when the transit has no vertical circle); and then, if two stars are observed at elongation, we have a check on the work.

16. *The Target.*—This may be a sort of box, in which a light may be placed. A narrow vertical slit should be cut, subtending an angle, at the instrument, from five to ten seconds of arc. This should be set as far from the instrument as convenient, as from a quarter of a mile to one mile. The width of slit desired may be computed, for any given angular width and distance, by remembering that the arc of one second is three-tenths of an inch for a mile radius. The target should be sufficiently distant to enable it to be seen with the stellar focus without appreciable parallax, as the instrument should not be refocused on the target. This target may be set on any convenient azimuth from the observation station, as upon one triangulation station when the observations are taken at another, thus obtaining directly the azimuth of this line.

17. *Illumination of Cross Wires.*—Various methods are used to illuminate the wires, the crudest of which is, perhaps, to hold a bull's-eye lantern so as to throw light down the telescope tube through the objective, taking care not to obstruct the line of sight.

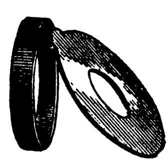

FIG. 9.

A very good reflector may be made from a piece of new tin, cut and bent as in Fig. 9.* The straight strip is bent about the object end of the telescope tube, leaving the annular elliptic piece projecting over in front. This is then bent to any desired angle, preferably about forty-five degrees, and turned so that an attendant can reflect light down the tube by illuminating the disk from a convenient position. This position should be so chosen, that the lantern may throw the light *from* the observer,

* The form of reflector shown in this figure is from the catalogue of W. & L. E. Gurley, Troy, N. Y.

rather than *towards* him. If the reflecting side of the disk be whitened, the effect is very good. The opening should be about three-fourths or seven-eighths inch in its shorter diameter, the longer diameter being such as to make its vertical projection equal to the shorter one. There is, of course, no necessity of limiting or of making true the outer edges of the disk.

18. *Instrumental Adjustments.* — The adjustments of the transit that are important in this work are those of the plate bubbles, the horizontal axis, and the line of collimation. The essential thing is, that the line of sight shall generate a vertical plane as the telescope revolves about its horizontal axis.

When the plate bubbles are in adjustment, the vertical axis may be made truly vertical. When the line of collimation is perpendicular to the horizontal axis of the telescope, the motion of the telescope about this axis will cause the line of collimation to generate *a plane.* When the ends of this horizontal axis are at the same elevation, this plane becomes a *vertical* plane.

All these adjustments must be carefully attended to, and the errors of adjustment made inappreciable, if a good result is to be obtained from a single observation.

It is preferable in all cases to make more than one observation, in order to eliminate these errors of adjustment so far as possible. There is no way of eliminating the errors due to the plate levels being out of adjustment. The other two sources of error may be eliminated, however, by a second observation, when the instrument is revolved a hundred and eighty degrees in azimuth, and the telescope transited. The same combined errors, from the want of adjustment in the line of collimation and the horizontal axis, then enter the result with the opposite sign, and therefore the mean of the two results will have these errors eliminated. These two observations may be made on the same star when at elongation. Thus, if the time of elongation is known, to the nearest minute, and if two sets of readings may be taken within four minutes either side of the time of elongation of Polaris, the error in azimuth will be less than one second. If the observation should be taken eight minutes

either side of the time of elongation, the error in azimuth is five seconds; and if the observation be taken fifteen minutes either side of the time of elongation, the error in azimuth is fifteen seconds of arc, — all this on the supposition that the assumed azimuth of the star when the observation is taken is its azimuth at elongation, as found from the equation on p. 82.

If, therefore, an error of five seconds of arc may be allowed, observations on the star may be made for a period of fifteen minutes; and this would enable several reversals to be made. If the following programme, covering twenty minutes of time, be followed, the instrumental errors will be well eliminated, and the star's mean place will be less than one second of arc from its azimuth at elongation. It will be observed that it brings the star readings all together at the time of elongation, gives four readings on the star and four on the mark, and allows two minutes for making a reading, and three minutes for a reversal and reading. Of course both verniers are to be read. If the time here given is not sufficient, it should be extended symmetrically before and after the time of elongation.

19. *Programme for observing for Azimuth on a Circumpolar Star at Elongation.*

Instrument.	Time of Observation.	Reading on.
Direct	10^m before elongation	Mark.
Reversed	7^m " "	"
"	4^m " "	Star.
"	2^m " "	"
Direct	2^m after elongation	"
"	4^m " "	"
"	7^m " "	Mark.
Reversed	10^m " "	"

The limits of error given above apply to observations on Polaris. If observations be made on 51 Cephei, these errors will be about twice as large.

The following approximate rule will be found convenient. It is based on the fact that the change in azimuth for a star near elongation varies as the square of the time from elongation. The rule may be applied to close circumpolar stars for a period of half an hour either side of the time of elongation, for latitude of 40°.

Rule: For reduction of Polaris to elongation, multiply the square of the time, in minutes, by 0.058; and this will be the reduction in seconds of arc.

For reduction of 51 Cephei to elongation, multiply the square of the time, in minutes, by 0.124; and this will be the reduction in seconds of arc.

The formula for reduction to elongation is

$$c = 112.5t^2 \sin 1'' \tan A,$$

where c = correction to observed azimuth in seconds of arc,
t = time from elongation in seconds of time, and
A = azimuth of the star at elongation.

$$\log 112.5 \sin 1'' = 6.7367274.$$

From this formula and that on p. 82 we may compute the co-efficients for the above approximate rules for any latitude. Thus, for latitude 30° we have azimuth of Polaris, 1885, 1° 30'.4, whence the co-efficient of reduction for elongation of Polaris in latitude 30° is found to be 0.052, and for latitude 50° it is 0.069.

For 51 Cephei, this co-efficient for latitude 30° is 0.110, and for latitude 50°, 0.148.

From the above data the corrections for an observation of a circumpolar star near elongation may be computed.

If azimuth be reckoned from the south point, as is common in topographical and other geodetic work, and if it increase in the direction S.W.N.E., then a star at western elongation has an azimuth of less than 180°, and at eastern elongation its azimuth is more than 180°.

The corrections to reduce to elongation, as above computed, should be added to the computed azimuth of the star at western elongation, and subtracted when at eastern elongation.

20. The local time cannot be conveniently observed except the meridian is known; but in this country railroad time is so generally accessible, and also so very accurate, that it is a very remote region indeed that is not reached by it. This, however, gives the time by hourly meridians; and to reduce this to local time involves a knowledge of the longitude of the place. This may be found from any good map. What is called Eastern time is five hours (75°) west from Greenwich; Central time being six hours (90°), Mountain time seven hours (105°), and Western time eight hours (120°), from Greenwich.

If the longitude is given from Washington, add 77° 3′ 0″.54 ($5^h\ 8^m\ 12^s$) to obtain longitude from Greenwich.

Knowing the longitude of the place, and the longitude for which the standard time is local time, the local time of the place is readily found.

CHAPTER IX.

PROJECTION OF MAPS, MAP LETTERING, AND TOPOGRAPHICAL SYMBOLS.

I. *Projection of Maps.*

1. The particular method that should be employed in representing portions of the earth's surface on a plane sheet or map depends, *first*, on the extent of the region to be represented; *second*, on the use to be made of the map or chart; and *third*, on the degree of accuracy desired.

Thus, a given kind of projection may suffice for a small region, but the approximation may become too inaccurate when extended over a large area. It is quite impossible to represent a spherical surface on a plane without sacrificing something in the accuracy of the relative distances, courses, or areas; and the use to which the chart is to be put must determine which of these conditions should be fulfilled at the expense of the others. A great many methods have been proposed and used for accomplishing various ends, some of which will be described.

2. *Rectangular Projection.* — In this method the meridians are all drawn as straight parallel lines; and the parallels are also straight, and at right angles with the meridians. A central meridian is drawn, and divided into minutes of latitude according to the value of these at that latitude as given in Table II. Through these points of division draw the parallels of longitude as right lines perpendicular to the central meridian. On

the central parallel lay off the minutes of longitude, according to their value for the given latitude, by Table II.; and through these points of division draw the other meridians parallel with the first.

The largest error here is in assuming the meridians to be parallel. For the latitude of 40°, two meridians a mile apart will converge at the rate of about a foot per mile. A knowledge of this fact will enable the draughtsman to decide when this method is sufficiently accurate for his purpose. Thus, for an area of ten miles square, the distortion at the extreme corners in longitude, with reference to the centre of the map as an origin of co-ordinates, will be about twenty-five feet. At the equator this method is strictly correct.

In this kind of projection, whether plotted from polar or rectangular co-ordinates, or from latitudes and longitudes, all straight lines of the survey, whether determined by triangulation, or run out by a transit on the ground, will be straight on the map; that is, the fore and back azimuth of a line is the same, or, in other words, a straight line on the drawing gives a constant angle with all the meridians.

This is the method to use on field sheets, where the survey has all been referred to a single meridian.

3. *Trapezoidal Projection.* — Here the meridians are made to converge properly, but both they and the parallels are straight lines. A central meridian is first drawn, and graduated to degrees or minutes; and through these points parallels are drawn, as before. Two of these parallels are selected; one about one-fourth the height of the map from the bottom, and the other the same distance from the top. These parallels are then subdivided, according to their respective latitudes, from Table II.; and through the corresponding points of division the remaining meridians are drawn as straight lines. The map is thus divided into a series of trapezoids. The parallels are perpendicular to but one of the meridians. The principal distortion comes from the parallels being drawn as straight lines, and amounts to about thirty-two feet in ten miles in

latitude 40°, and is nearly proportional to the square of the distance east or west from the central meridian.

The work should be plotted from computed latitudes and longitudes. The method is adapted to a scheme which has a system of triangulation for its basis, the geodetic position of the stations having been determined. These conditions would be fulfilled in a State topographical or geological survey for the separate sheets, each sheet covering an area of not more than twenty-five miles square.

4. *The Simple Conic Projection.* — In this projection, points on a spherical surface are first projected upon the surface of a tangent cone, and then this conical surface is developed into the plane of the map. The apex of the cone is taken in the axis of the earth extended, at such an altitude that the cone becomes tangent to the earth's surface at the middle parallel of the map. When this conical surface is developed into a plane, the meridians are straight lines converging to the apex of the cone, and the parallels are arcs of concentric circles about the apex as the common centre.

The sheet is laid out as follows: Draw a central meridian, and graduate it to degrees or minutes, according to their true values as given in Table II. Through these points of division draw parallel circular arcs, using the apex of the cone as the common centre. For values of the length of the side of the tangent cone, which is the length of the central parallel above, see Table II. The rectangular co-ordinates of points in these curves are also given in the same table.

On the middle parallel of the map the degrees or minutes of longitude are laid off, and through these are drawn the remaining meridians as straight lines radiating from the apex of the tangent cone.

It will be seen that the latitudes are correctly laid off, and the degrees of longitude will be sufficiently accurate for a map covering an area of several hundred miles square. The meridians and parallels are at right angles.

In this projection, the degrees of longitude on all parallels,

except the middle one, are too great; and therefore the area represented on the map is greater than the corresponding area on the sphere.

The chart should be plotted from computed latitudes and longitudes.

5. *De l'Isle's Conic Projection.* — This is very similar to the above, except that two parallels, one at one-fourth, and one at three-fourths, the height of the map, are properly graduated, and the meridians drawn as straight lines through these points of division. The parallels are drawn as concentric circles, as in the simple conic projection. This is therefore but a combination of the second and third methods, and is more accurate than either of them. The cone here is no longer tangent, but intersects the sphere in the two graduated parallels. In this case the region between the parallels of intersection is shown too small, and that outside these lines is shown too large; so that the area of the whole map will correspond very closely to the corresponding area on the sphere. When these parallels are so selected that these areas will be to each other exactly as the scale of the drawing, then it is called "Murdoch's projection."

6. *Bonne's Projection.* — This differs from the simple conic in this: that all the parallels are properly graduated, and the meridians drawn to connect the corresponding points of division in the parallels. These latter are, however, still concentric circles. The meridians are at right angles to the parallels only in the middle portion of the map. The same scale applies to all parts of the chart. There is a slight distortion at the extreme corners, from the parallels being arcs of concentric circles. The proportionate equality of areas is preserved. A rhumb line appears as a curve; but, when once drawn, its length may be properly scaled.

It will be noted that the last three methods involve the use of but one tangent or intersecting cone.

7. *The Polyconic Projection.* — For very large areas it is preferable to have each parallel the development of the base

of a cone tangent in the plane of the given parallel. This projection differs from Bonne's only in the fact that the parallels are no longer concentric arcs, but each is drawn with a radius equal to the side of the cone which is tangent at that latitude. These, of course, decrease towards the pole; and therefore the parallels diverge from each other towards the edge of the chart. The result of this is, that a degree of latitude at the side of the map is not equal to a degree on the central meridian; or, in other words, the same scale cannot be applied to all parts of the map. These defects appear, however, only on maps representing very large areas. The whole of North America could be represented by this method without any material distortion.

This method of projection is exclusively used on the United States Coast and Geodetic Survey, and for all other maps and charts of large areas in this country. Extensive tables are published by the War and Navy Departments, and also by the Coast Survey, to facilitate the projection of maps by the polyconic system. Table II. gives in a condensed form the rectangular co-ordinates of the points of intersection of the parallels and meridians referred to the intersection of the several parallels with the central meridian as the several origins.

8. *Formulæ Used in the Projection of Maps.* — The fundamental relations on which the method of polyconic projection rests, are given in the following formulæ : —

Normal, being the radius of curvature of a section perpendicular to the meridian at a given point . . . . $N = \dfrac{R_e}{(1 - e^2 \sin^2 L)^{\frac{1}{2}}}$, (1)

where R_e is the equational radius,
e is the eccentricity,
and L is the latitude.

Radius of the meridian . . . . $R_m = N^3 \dfrac{(1 - e^2)}{R_e^2}$. (2)

Radius of the parallel . . . . . $R_p = N \cos L$. (3)

Degree of the meridian . . . . $D_m = \frac{\pi}{180} R_m$ (4)

$= 3600\, R_m \sin 1''.$

Degree of the parallel. . . . . $D_p = \frac{\pi}{180} R_p$ (5)

$= 3600\, R_p \sin 1''.$

Radius of the developed parallel, or side of tangent cone . . . . $r = N \cot L.$ (6)

If n be any arc of a parallel, in degrees, or any difference of longitude from the central meridian of the drawing, and if θ be the corresponding angle, in degrees, at the vertex of the tangent cone, subtended by the developed parallel, then the length of the given arc will be

$$nD_p = \frac{\pi}{180} nR_p = \frac{\pi}{180} nN \cos L.$$

But the arc

$$\left.\begin{aligned} nD_p &= \frac{\pi}{180} nR_p = \frac{\pi}{180} nN \cos L. \\ nD_p &= \frac{\pi}{180} \theta r = \frac{\pi}{180} \theta N \cot L, \end{aligned}\right\} \quad (7)$$

whence

Angle of the developed parallel $\theta = n \sin L.$ (8)

Since the developed parallels are circular arcs, the rectangular co-ordinates of any point an angular distance of θ from the central meridian is

$$\left.\begin{aligned} &\text{Meridian distance . . } d_m = x = r \sin \theta. \\ &\text{Divergence of parallels, } d_p = y = r \operatorname{vers} \theta \\ &\qquad = x \tan \tfrac{1}{2}\theta. \end{aligned}\right\} \quad (9)$$

For arcs of small extent, the parallel may be considered coincident with its chord; but the angle between the axis of x and the chord is $\frac{1}{2}\theta$. If, then, the length of the arc, which is nDp, be represented by the chord, we may write

and

$$\left.\begin{aligned} d_m &= \text{meridian distance} = x = nD_p \cos \tfrac{1}{2}\theta, \\ d_p &= \text{divergence of parallels} = y = nD_p \sin \tfrac{1}{2}\theta. \end{aligned}\right\} \quad (10)$$

If, now, d_{m1} = meridian distance for 1 degree of longitude,
and d_{mn} = meridian distance for n degrees of longitude,
we have

$$\frac{d_{mn}}{d_{m1}} = \frac{nD_p \cos \frac{1}{2} \theta_n}{D_p \cos \frac{1}{2} \theta_1}.$$

But $\theta = n \sin L$, so that $\theta_1 = 1° \times \sin L = 38'$ for latitude 40°. Therefore

$$\cos \tfrac{1}{2} \theta_1 = \cos 19' = 1, \text{ nearly};$$

so that

$$\frac{d_{mn}}{d_{m1}} = n \cos \tfrac{1}{2} (n \sin L), \text{ nearly.} \qquad (11)$$

For $L = 30°$, we have $\sin L = \frac{1}{2}$. Therefore, for latitude 30°,

$$\frac{d_{mn}}{d_{m1}} = n \cos \tfrac{1}{4} n = n \cos (0.25n) \text{ nearly.}$$

If we have obtained the meridian distance, d_m, for 1 degree of longitude, and wish to obtain it for n degrees in latitude 30°, we have but to multiply the distance for 1 degree by $n \cos (0.25n)$.

9. In Table II. the meridian distances are given, at various latitudes, for a difference of longitude of one degree. To find the meridian distance for any number of degrees or parts of degrees, multiply the distance for one degree by the factor there given for the given latitude. The factor given in the table for latitude 30° is $n \cos (0.288n)$, in place of $n \cos (0.25n)$, as obtained above. The difference is a correction which has been introduced to compensate the error made in assuming that the chord was equal in length to its arc. The corrected factors enable the table to be used without material error up to 25 degrees longitude either side of the central meridian.

To obtain the divergence of the parallels for differences of longitude more or less than one degree, multiply the divergence for one degree by the square of the number of degrees. It is evident that this rule is based on the assumption that the arc of

the developed parallel is a parabola, and so it may be considered for a distance of 25 degrees either side of the central meridian between the latitudes 30° and 50° without material error.

If the whole of the United States were projected by this table, using the factors given, to a scale of 1 to 1,500,000, thus giving a map some 8 by 10 feet, the maximum deviation of the meridians and parallels from their true positions (which would be at the upper corners) would be but about 0.02 inch.

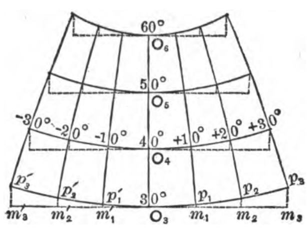

FIG. 10.

Thus, for a map of this size, covering 20 degrees of latitude and 50 degrees of longitude, the geodetic lines would have their true position within the width of a fine pencil line, by the use of Table II. Fig. 10 will illustrate the use of the table in projecting a map by the polyconic method. The map covers 30 degrees in latitude (30° to 60°) and 60 degrees in longitude. The straight line $O_3\ O_6$ is first drawn through the centre of the map, and graduated according to the lengths of one degree of latitude, as given in the second column of Table II. Through these points of division the lines $m_3'\ m_3$, are drawn in pencil at right angles to the central meridian. On these lines the points m_1, m_2, etc., are laid off by the aid of the first part of Table II. This table gives the meridian distances when n is less than one degree, as well as when n is greater. From the points m_1, m_2, etc., the divergence of the parallels is laid off above the lines O, m, by the aid of the second portion of Table II., thus obtaining the positions of the points p_1, p_2, etc. The points p mark the intersection of the meridians and parallels; and these may be drawn as straight lines between these points, provided a

sufficient number of such points have been located. The map is then to be plotted upon the chart from computed latitudes and longitudes.

10. *Summary.* — We have seen that there are, in general, two ways of plotting a map or chart, and two corresponding uses to which it may be put: —

First, We may plot by a system of plane co-ordinates, either polar (azimuth and distance) or rectangular (latitudes and departures). This gives a map from which distance, azimuth (referred to the meridian of the map), and areas are correctly determined.

Second, We may plot the map by computed latitudes and longitudes, and determine from it the relative position of points in terms of their latitude and longitude.

The first system is adapted to small field sheets and detail charts for which the notes were taken by referring all points to a single point and meridian. For this purpose the system of rectangular projection should be selected, as long as the area of the chart is not more than about one hundred square miles. If it be larger than this, the trapezoidal system should be used. In case this is done, the work is still plotted as before, provided it has all been referred to a given meridian in the field work, and then converging meridians are drawn as described above. From such a chart, not only the azimuth (referred to the central meridian) and distance may be determined, but the correct longitude and nearly correct latitude are given.

In the case of topographical charts, based on a system of triangulation, each sheet is referred to a meridian passing through a triangulation station on that sheet, or near to it, and the rectangular system used.

11. In the case of a survey of a long and narrow belt, as for a river, railroad, or canal, if the survey was based on a system of triangulation, the convergence of meridians has been looked after in the computation of the geodetic positions of these stations, and each sheet is plotted by the rectangular

system, being referred to the meridian through the adjacent trangulation station. When many of these are combined into a single map on a small scale, then they must be plotted on the condensed map by latitudes and longitudes, these being taken from the small rectangular projections, and plotted on the reduced chart in polyconic projection; the meridians and parallels having been laid out as shown above.

In case the belt extends mostly east and west, and is not based on a triangulation scheme, then observations for azimuth should be made as often as every fifty miles. It will not do to run on a given azimuth for this distance, however; for there has been a change in the direction of the parallel (or meridian) in this distance, in latitude 40°, of about 40 minutes. According to the accuracy with which the work is done, therefore, when running wholly by back azimuths, the setting of the instrument must be changed. Thus, if in going 1 degree (53 miles), east or west, in latitude 40°, the meridian has shifted 40′, then in going 13 miles east or west, the meridian has changed 10′; and this is surely a sufficiently large correction to make it worth while to apply it.

When running west, this correction is applied in the direction of the hands of a watch, and when running east, in the opposite direction; that is, having run west 13 miles by back azimuth, then the pointing which appears north is really 10′ west of north, and the telescope must be shifted 10′ around to the right.

If the azimuth be corrected in this way, a survey can be carried by back azimuth an indefinite distance, and still have the entire survey referred to the true meridian.

12. The angle of convergence of meridians is the angle θ in the equations given in the above formula. Then

$$\theta = n \sin L,$$

where n is the angular change in degrees of longitude, and L is the latitude of the place.

For $L = 30°$, $\sin L = \frac{1}{2}$; or, in latitude 30° a change of longitude of one degree changes the direction of the meridian by 30 minutes.

For $L = 40°$, $\sin L = 0.643$; or, a change of longitude of one degree changes the direction of the meridian by 0.643 of 60 minutes, or 38.6 minutes, being approximately 40 minutes.

For $L = 50°$, $\sin L = 0.766$; or, in going east or west one degree, the meridian changes 0.766×60 minutes $= 46$ minutes, or approximately 50 minutes.

Therefore we may have the approximate rule, that a change of longitude of one degree changes the azimuth by as many minutes as equals the degrees of latitude of the place. This rule gives results very near the truth between plus and minus 40° latitude, that is, over an equatorial belt 80 degrees in width.

II. *Map Lettering and Topographical Symbols.*

13. The best-drawn map may have its appearance ruined by the poor skill or bad taste displayed in the lettering. The letters should be simple, neat, and dignified in appearance, and have their size properly proportioned to the subject. The map is lettered before the topographical symbols are drawn. When a map is drawn for popular display, some ornamentation may be allowed in the title; but even then, the lettering on the map itself should be plain and simple. When the map is for official or professional use, even the title should be made plain.

On Plate II. are given several sets of alphabets which are well adapted to map work. Of course the size should vary according to the scale of the map and the subject, as shown on Plate III. It is a good rule to make all words connected with water in Italics. The small letters in stump writing will be found very useful, and these should be practised thoroughly. The Italic capitals go with these small letters also.

14. In place of the system of letters above described, and

which has heretofore been almost exclusively used for mapping-purposes, a new system, called "round writing," may be used. A text-book on this method, by F. Soennecken, is published by Messrs. Kueffel & Esser, New York. This work is done with blunt pens, all lines being made with a single stroke. It looks well when well done, and requires but a small fraction of the time required to make the ordinary letters. For working drawings and field maps it is especially adapted.

15. In topographical representation, where elevations have been taken sufficiently numerous and accurate, the outline of the ground should be represented by contours rather than by hachures, or hill shading, which simply gives an approximate notion of the slope of the ground, but no indication of its actual elevation. Where the ground has so steep a slope that the contour lines would fall one upon another, it is well here to put in shading-lines, as shown on Plate I. The water surfaces and streams may be water-lined in blue, or left white. The contour lines over alluvial ground should be in brown (crimson and burnt sienna), while those over rocky and barren ground should be in black. This is a very simple and effective method of showing the character of the soil.

The practices of the government surveys should be followed in the matter of conventional surface representation, such as meadow, swamp, woodland, prairie, cane-brake, etc., with all their varieties. Some of these are given in the United States Coast Survey Report for 1879 and 1883, while Plate I. shows most of those used on the Mississippi River Survey. On this the contours are all in black, for the purpose of photo-lithographing.

TABLE I.*

Minutes.	0° Hor. Dist.	0° Diff. Elev.	1° Hor. Dist.	1° Diff. Elev.	2° Hor. Dist.	2° Diff. Elev.	3° Hor. Dist.	3° Diff. Elev.
0 . .	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2 . .	"	0.06	"	1.80	99.87	3.55	99.72	5.28
4 . .	"	0.12	"	1.86	"	3.60	99.71	5.34
6 . .	"	0.17	99.96	1.92	"	3.66	"	5.40
8 . .	"	0.23	"	1.98	99.86	3.72	99.70	5.46
10 . .	"	0.29	"	2.04	"	3.78	99.69	5.52
12 . .	"	0.35	"	2.09	99.85	3.84	"	5.57
14 . .	"	0.41	99.95	2.15	"	3.90	99.68	5.63
16 . .	"	0.47	"	2.21	99.84	3.95	"	5.69
18 . .	"	0.52	"	2.27	"	4.01	99.67	5.75
20 . .	"	0.58	"	2.33	99.83	4.07	99.66	5.80
22 . .	"	0.64	99.94	2.38	"	4.13	"	5.86
24 . .	"	0.70	"	2.44	99.82	4.18	99.65	5.92
26 . .	99.99	0.76	"	2.50	"	4.24	99.64	5.98
28 . .	"	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30 . .	"	0.87	"	2.62	"	4.36	"	6.09
32 . .	"	0.93	"	2.67	99.80	4.42	99.62	6.15
34 . .	"	0.99	"	2.73	"	4.48	"	6.21
36 . .	"	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38 . .	"	1.11	"	2.85	"	4.59	99.60	6.33
40 . .	"	1.16	"	2.91	99.78	4.65	99.59	6.38
42 . .	"	1.22	99.91	2.97	"	4.71	"	6.44
44 . .	99.98	1.28	"	3.02	99.77	4.76	99.58	6.50
46 . .	"	1.34	99.90	3.08	"	4.82	99.57	6.56
48 . .	"	1.40	"	3.14	99.76	4.88	99.56	6.61
50 . .	"	1.45	"	3.20	"	4.94	"	6.67
52 . .	"	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54 . .	"	1.57	"	3.31	99.74	5.05	99.54	6.78
56 . .	99.97	1.63	"	3.37	"	5.11	99.53	6.84
58 . .	"	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60 . .	"	1.74	"	3.49	"	5.23	99.51	6.96
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$c = 1.25$	1 25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

* This table was computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania.

TABLE I.—*Continued.*

Minutes.	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2 . .	"	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4 . .	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6 . .	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8 . .	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10 . .	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12 . .	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14 . .	"	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16 . .	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18 . .	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20 . .	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22 . .	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24 . .	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26 . .	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28 . .	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30 . .	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32 . .	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34 . .	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36 . .	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38 . .	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40 . .	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42 . .	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44 . .	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46 . .	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48 . .	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50 . .	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52 . .	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54 . .	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56 . .	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58 . .	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60 . .	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c = 0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$c = 1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$c = 1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE I.—*Continued.*

Minutes.	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2 . .	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4 . .	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6 . .	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8 . .	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10 . .	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12 . .	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14 . .	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16 . .	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18 . .	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20 . .	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22 . .	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24 . .	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26 . .	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28 . .	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30 . .	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32 . .	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34 . .	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36 . .	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38 . .	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40 . .	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42 . .	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44 . .	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46 . .	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48 . .	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50 . .	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52 . .	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54 . .	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56 . .	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58 . .	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60 . .	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$c = 0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$c = 1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$c = 1.25$	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE I. — *Continued.*

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2 . .	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4 . .	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6 . .	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8 . .	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10 . .	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12 . .	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14 . .	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16 . .	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18 . .	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20 . .	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22 . .	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24 . .	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26 . .	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28 . .	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30 . .	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32 . .	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34 . .	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36 . .	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38 . .	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40 . .	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42 . .	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44 . .	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46 . .	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48 . .	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50 . .	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52 . .	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54 . .	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56 . .	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58 . .	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60 . .	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c = 0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$c = 1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$c = 1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE I.—*Continued.*

Minutes.	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2 . .	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4 . .	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6 . .	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8 . .	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10 . .	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12 . .	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14 . .	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16 . .	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18 . .	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20 . .	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22 . .	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24 . .	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26 . .	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28 . .	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30 . .	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32 . .	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34 . .	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36 . .	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38 . .	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40 . .	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42 . .	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44 . .	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46 . .	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48 . .	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50 . .	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52 . .	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54 . .	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56 . .	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58 . .	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60 . .	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
$c = 0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$c = 1.00$	0.86	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$c = 1.25$	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE I.—*Continued.*

Minutes.	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2 . .	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4 . .	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6 . .	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8 . .	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10 . .	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12 . .	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14 . .	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16 . .	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18 . .	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20 . .	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22 . .	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24 . .	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26 . .	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28 . .	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30 . .	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32 . .	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34 . .	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36 . .	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38 . .	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40 . .	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42 . .	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44 . .	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46 . .	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48 . .	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50 . .	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52 . .	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54 . .	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56 . .	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58 . .	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60 . .	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c = 0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$c = 1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$c = 1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE I.—*Continued.*

Minutes.	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev	Hor. Dist.	Diff. Elev.
0 . .	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2 . .	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4 . .	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6 . .	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8 . .	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10 . .	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12 . .	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14 . .	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16 . .	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18 . .	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20 . .	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22 . .	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24 . .	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26 . .	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28 . .	82.85	37.70	81.51	38.62	80.14	39.90	78.73	40.92
30 . .	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32 . .	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34 . .	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36 . .	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38 . .	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40 . .	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42 . .	82.54	37.96	81.19	39.08	79 81	40.14	78.39	41.16
44 . .	82.49	38.00	81.15	39.11	79 76	40.18	78.34	41.19
46 . .	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48 . .	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50 . .	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52 . .	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54 . .	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56 . .	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58 . .	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60 . .	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c = 0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$c = 1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$c = 1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE I.— *Concluded.*

Minutes.	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	77.96	41.45	76.50	42.40	75.00	43.30
2 . .	77.91	41.48	76.45	42.43	74.95	43.33
4 . .	77.86	41.52	76.40	42.46	74.90	43.36
6 . .	77.81	41.55	76.35	42.49	74.85	43.39
8 . .	77.77	41.58	76.30	42.53	74.80	43.42
10 . .	77.72	41.61	76.25	42.56	74.75	43.45
12 . .	77.67	41.65	76.20	42.59	74.70	43.47
14 . .	77.62	41.68	76.15	42.62	74.65	43.50
16 . .	77.57	41.71	76.10	42.65	74.60	43.53
18 . .	77.52	41.74	76.05	42.68	74.55	43.56
20 . .	77.48	41.77	76.00	42.71	74.49	43.59
22 . .	77.42	41.81	75.95	42.74	74.44	43.62
24 . .	77.38	41.84	75.90	42.77	74.39	43.65
26 . .	77.33	41.87	75.85	42.80	74.34	43.67
28 . .	77.28	41.90	75.80	42.83	74.29	43.70
30 . .	77.23	41.93	75.75	42.86	74.24	43.73
32 . .	77.18	41.97	75.70	42.89	74.19	43.76
34 . .	77.13	42.00	75.65	42.92	74.14	43.79
36 . .	77.09	42.03	75.60	42.95	74.09	43.82
38 . .	77.04	42.06	75.55	42.98	74.04	43.84
40 . .	76.99	42.09	75.50	43.01	73.99	43.87
42 . .	76.94	42.12	75.45	43.04	73.93	43.90
44 . .	76.89	42.15	75.40	43.07	73.88	43.93
46 . .	76.84	42.19	75.35	43.10	73.83	43.95
48 . .	76.79	42.22	75.30	43.13	73.78	43.98
50 . .	76.74	42.25	75.25	43.16	73.73	44.01
52 . .	76.69	42.28	75.20	43.18	73.68	44.04
54 . .	76.64	42.31	75.15	43.21	73.63	44.07
56 . .	76.59	42.34	75.10	43.24	73.58	44.09
58 . .	76.55	42.37	75.05	43.27	73.52	44.12
60 . .	76.50	42.40	75.00	43.30	73.47	44.15
$c = 0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$c = 1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$c = 1.25$	1.10	0.60	1.09	0.62	1.08	0.64

TABLE II.

CO-ORDINATES OF POINTS OF INTERSECTION OF PARALLELS AND MERIDIANS IN POLYCONIC PROJECTION.

Latitude.	Length of 1° Latitude, in Statute Miles.	Length of Side of Tangent Cone, in Statute Miles.	Meridian Distances for 1° Longitude.				Divergence of Parallels for 1° Longitude.			
			In Yards.	In Metres.	In Miles.	Factor.	In Yards.	In Metres.	In Miles.	Factor.
30°	68.875	6869	105507	96476	59.95	$n \cos(0.288n^\circ)$	460.4	421.0	0.2617	n^2
32°	68.897	6348	103327	94481	58.71	$n \cos(0.304n^\circ)$	477.8	436.8	0.2715	n^2
34°	68.918	5881	101022	92373	57.40	$n \cos(0.320n^\circ)$	493.0	450.7	0.2800	n^2
36°	68.941	5461	98593	90152	56.02	$n \cos(0.337n^\circ)$	505.7	462.4	0.2873	n^2
38°	68.964	5079	96044	87822	54.57	$n \cos(0.353n^\circ)$	516.0	471.8	0.2932	n^2
40°	68.987	4729	93377	85383	53.06	$n \cos(0.369n^\circ)$	523.8	479.0	0.2976	n^2
42°	69.011	4408	90596	82840	51.48	$n \cos(0.386n^\circ)$	529.0	483.8	0.3006	n^2
44°	69.036	4110	87704	80197	49.83	$n \cos(0.402n^\circ)$	531.7	486.2	0.3022	n^2
46°	69.060	3833	84704	77452	48.13	$n \cos(0.418n^\circ)$	531.7	486.2	0.3022	n^2
48°	69.084	3575	81601	74615	46.37	$n \cos(0.435n^\circ)$	529.2	484.0	0.3007	n^2
50°	69.108	3332	78398	71686	44.54	$n \cos(0.451n^\circ)$	524.1	479.2	0.2978	n^2

n = number degrees of longitude between the given meridian and the prime meridian of the map.

ROMAN CAPITALS.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Roman Small.

a b c d e f g h i j k l m n o p q r s t u v w x y z

ITALIC CAPITALS.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Italic Small.

a b c d e f g h i j k l m n o p q r s t u v w x y z

Stump Writing.

a b c d e f g h i j k l m n o p q r s t u v w x y z

I II III IV V VI VII VIII IX X — 50 L, 100 C, 500 D, 1000 M — 0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

MAP LETTERING.

CAPITALS, FORTRESSES of 1st Class, CHAIN of MOUNTAINS,
Oceans and Frontiers.

CITIES, BOUNDARIES of a STATE, FORTRESSES of 2d Rank,
Seas, Great Lakes, Mountain Branches.

FORTRESSES of 3d Rank, *STREAMS*, *NAVIGABLE RIVERS*, PLATEAUS,
Glaciers and Forests of large extensions.

Towns, Forts, *Navigable Canals,* Parts of Mountains, Summits, Forks, Abysses, *Glaciers*
of less extensions, Boundaries of Counties.

Large Villages, Fortified Passes, Rivers navigable by Rafts, Lakes, Ponds, Forests.

Smaller Villages; Hamlets; Rivers not navigable; Creeks, Brooks, etc., not fordable.

Isolated Churches, Convents, Castles, Factories, Farmhouses, Mansions, and other smaller objects.

Note. — *All names and words connected with Water to be slanting Capitals and Italics.*